Topology Doctoral Preliminary Examination

May 2019

Give complete proofs and computations for all your answers.

1. Determine which of the following topological spaces are (a) homeomorphic and (b) homotopy equivalent to each other, and which ones are not: S^n, $S^n \setminus \{*\}$, $T^n \setminus \{*\}$, for all values of $n \geq 1$. Here $*$ denotes some arbitrary point in the space and $T^n = (S^1)^n$ is the n-dimensional torus.

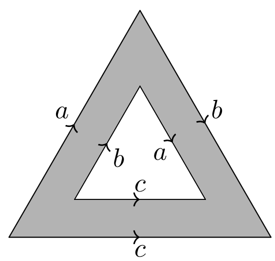
2. Consider a donut, i.e., a torus (orientable genus 1 surface) together with its interior. A worm starts at some point on the surface, eats through the donut leaving behind an empty, nonselfintersecting tunnel, and exits at some other point on the surface. Compute the homology with coefficients in $\mathbf{Z}$ and the fundamental group for at least one choice of a tunnel.

3. A point is removed from a nonorientable surface with 3 crosscaps. Compute the fundamental group of the resulting space. Does this space admit a connected covering of degree $n \geq 1$? Does it admit a connected Galois covering of degree $n \geq 1$? Does it admit a connected covering with S_3 (the symmetric group of order 3) as its deck transformation group?

4. Classify all local systems with typical fiber $\mathbf{Z}$ on a Klein bottle and compute the twisted homology with coefficients in these local systems.

5. Compute the cohomology ring and Poincaré duality isomorphism of the manifold $S^1 \times S^2$ with coefficients in an arbitrary commutative unital ring A.

6. Consider the following manifold (a surface) given by the area between the two triangles, with labeled sides glued when the labels are the same:



Is this manifold orientable or not? Compute its cohomology ring and Poincaré duality isomorphism. What surface in the classification of surfaces (recall its statement) does this surface correspond to?