1. Determine which of the following topological spaces are (a) homeomorphic and (b) homotopy equivalent to each other, and which ones are not: $S^n$, $S^n \setminus \{\ast\}$, $T^n \setminus \{\ast\}$, for all values of $n \geq 1$. Here $\ast$ denotes some arbitrary point in the space and $T^n = (S^1)^n$ is the $n$-dimensional torus.

2. Consider a donut, i.e., a torus (orientable genus 1 surface) together with its interior. A worm starts at some point on the surface, eats through the donut leaving behind an empty, nonself-intersecting tunnel, and exits at some other point on the surface. Compute the homology with coefficients in $\mathbb{Z}$ and the fundamental group for at least one choice of a tunnel.

3. A point is removed from a nonorientable surface with 3 crosscaps. Compute the fundamental group of the resulting space. Does this space admit a connected covering of degree $n \geq 1$? Does it admit a connected Galois covering of degree $n \geq 1$? Does it admit a connected covering with $S_3$ (the symmetric group of order 3) as its deck transformation group?

4. Classify all local systems with typical fiber $\mathbb{Z}$ on a Klein bottle and compute the twisted homology with coefficients in these local systems.

5. Compute the cohomology ring and Poincaré duality isomorphism of the manifold $S^1 \times S^2$ with coefficients in an arbitrary commutative unital ring $A$.

6. Consider the following manifold (a surface) given by the area between the two triangles, with labeled sides glued when the labels are the same:

Is this manifold orientable or not? Compute its cohomology ring and Poincaré duality isomorphism. What surface in the classification of surfaces (recall its statement) does this surface correspond to?