

Topology Doctoral Preliminary Examination Topics

1 Simplicial sets

- Simplicial sets, simplicial maps, generators and relations for simplicial sets.
- Limits and colimits of simplicial sets, function complexes, nerve-realization adjunction.
- The Dold–Kan correspondence, Eilenberg–Zilber theorem.

2 Homology and cohomology

- Simplicial homology and cohomology with coefficients in an abelian group.
- Cup and cap products, cohomology ring and homology module.
- Invariance under homotopies and homotopy equivalences.

3 Manifolds

- Manifolds.
- Orientation, fundamental class.
- Local systems.
- Poincaré duality.

4 Homotopy

- Homotopy groups, Hurewicz homomorphism, Hurewicz theorem.
- Homotopy groups of spheres. Degree and Hopf invariant.
- Fundamental groupoids and coverings, Seifert–van Kampen theorem, Galois theory of coverings.
- Kan complexes, Kan’s fibrant replacement functor, weak equivalences.
- Whitehead theorem.
- Homotopy limits and colimits.
- Generalized homology and cohomology theories, spectra.

5 Point-set theory

- Topological spaces, continuous maps, bases and subbases, metric spaces, separability.
- Subspaces, quotients, limits and colimits of topological spaces.
- Compact spaces, Hausdorff spaces. Connected spaces. Topological manifolds.
- Singular simplicial sets of topological spaces and geometric realizations of simplicial sets.
- Brouwer fixed point theorem.
- Simplicial approximation theorem, Lefschetz fixed point theorem.
- Nerve and hypernerve theorem.

6 Local homotopy theory

- Sheaves and simplicial presheaves on topological spaces and sites.
- Homotopy descent, Verdier hypercovering theorem.
- Mayer–Vietoris properties.