Topology Doctoral Preliminary Examination Topics

1 Simplicial sets
   - Simplicial sets, simplicial maps, generators and relations for simplicial sets.
   - Limits and colimits of simplicial sets, function complexes, nerve-realization adjunction.
   - The Dold–Kan correspondence, Eilenberg–Zilber theorem.

2 Homology and cohomology
   - Simplicial homology and cohomology with coefficients in an abelian group.
   - Cup and cap products, cohomology ring and homology module.
   - Invariance under homotopies and homotopy equivalences.

3 Manifolds
   - Manifolds.
   - Orientation, fundamental class.
   - Local systems.
   - Poincaré duality.

4 Homotopy
   - Homotopy groups, Hurewicz homomorphism, Hurewicz theorem.
   - Homotopy groups of spheres. Degree and Hopf invariant.
   - Fundamental groupoids and coverings, Seifert–van Kampen theorem, Galois theory of coverings.
   - Kan complexes, Kan’s fibrant replacement functor, weak equivalences.
   - Whitehead theorem.
   - Homotopy limits and colimits.
   - Generalized homology and cohomology theories, spectra.

5 Point-set theory
   - Topological spaces, continuous maps, bases and subbases, metric spaces, separability.
   - Subspaces, quotients, limits and colimits of topological spaces.
   - Singular simplicial sets of topological spaces and geometric realizations of simplicial sets.
   - Brouwer fixed point theorem.
   - Simplicial approximation theorem, Lefschetz fixed point theorem.
   - Nerve and hypernerve theorem.

6 Local homotopy theory
   - Sheaves and simplicial presheaves on topological spaces and sites.
   - Homotopy descent, Verdier hypercovering theorem.
   - Mayer–Vietoris properties.