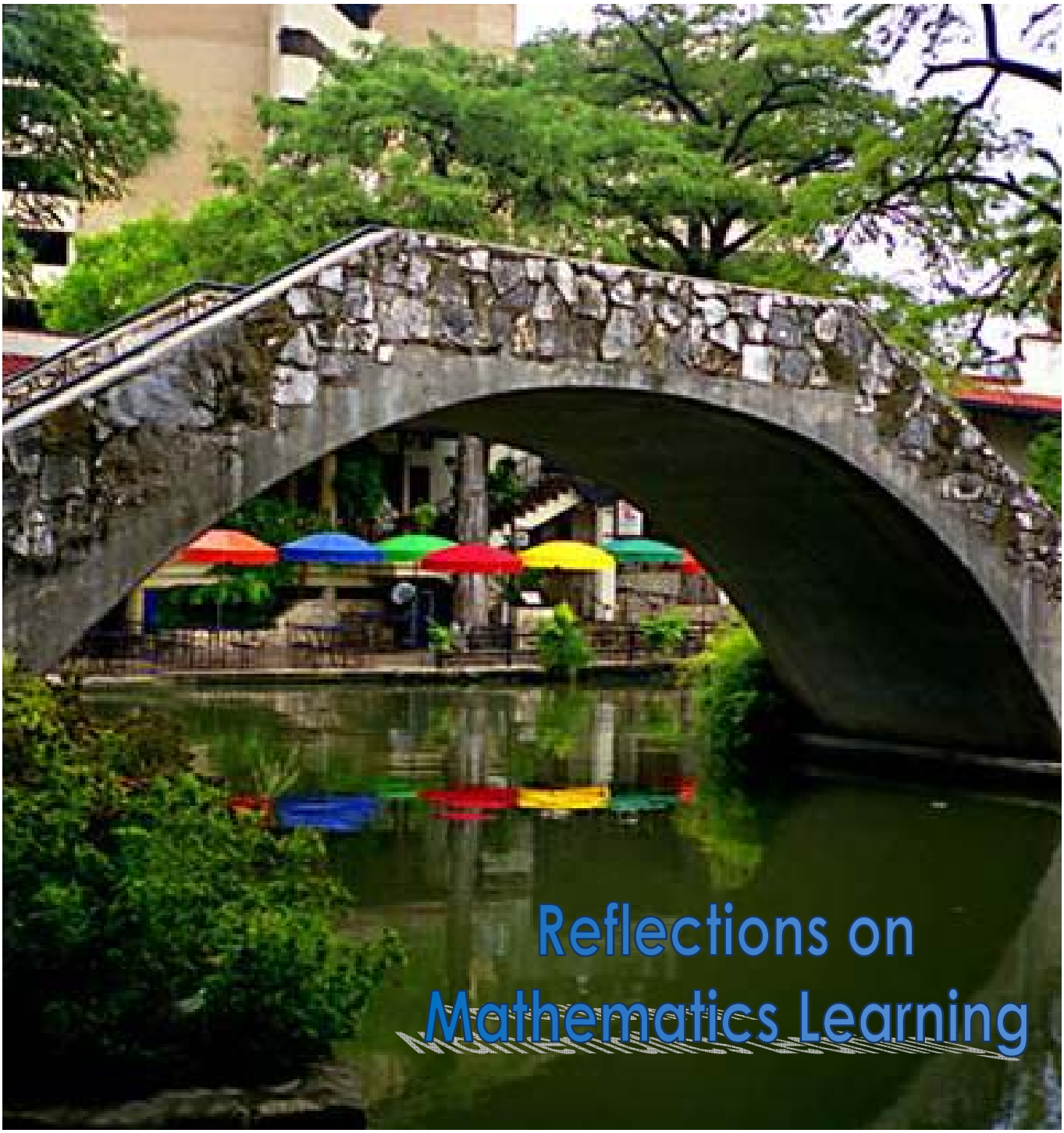
Proceedings for the 41th Annual Meeting of the Research Council on Mathematics Learning

February 27 – March 1, 2014 San Antonio, Texas



Mathematics Teaching Methods and Practice

Teachers of Mathematics

Mathematics Learning

RIGOROUS MATH COURSES FOR MIDDLE-SCHOOL MATH TEACHERS

Gary Harris
Texas Tech University
gary.harris@ttu.edu

Tara Stevens
Texas Tech University
tara.stevens@ttu.edu

Raegan Higgins
Texas Tech University
raegan.higgins@ttu.edu

Zeniada Aguirre-Munoz
Texas Tech University
z.aguirre@ttu.edu

Xun Liu
Texas Tech University
xun.liu@ttu.edu

We describe our five-year professional development project targeting middle-school math teachers. A primary focus of the project is providing the teachers with a deep conceptual understanding of the mathematics taught in the middle grades. Our analyses to date indicate a positive impact on the teachers' conceptual knowledge, their math knowledge for teaching, and their self-efficacy for teaching mathematics to diverse student populations. Preliminary findings also suggest a positive impact on teachers' classroom practices.

It is generally accepted that teachers of mathematics need to possess a deep conceptual understanding of the elementary mathematics they are teaching (Ma, 1999; CBMS, 2012) and need to have special knowledge related to the teaching of mathematics, often referred to as mathematics knowledge for teaching or MKT (Shulman, 1987; Ball et al, 2008). In addition, effective teachers need a strong belief in their ability to teach mathematics to diverse student populations (Hernandez et al., 2008; Moschkovich, 2002). In this paper we describe a five-year professional development project designed to increase the conceptual understanding, the MKT, and the math teaching self-efficacy of in-service middle-school math teachers in a large rural area of West Texas. We then provide a survey of the results we have obtained in relation to these teacher outcomes. Finally, we describe our continuing efforts to further document the long-term project effects on not only these teacher outcomes but also on teachers' classroom practice.

Project Description

The West Texas Middle School Math Partnership, WTMSMP, is a five-year professional development (PD) project funded by the National Science Foundation Math Science Partnership program, beginning January 1, 2009 and ending December 31, 2013. During this time period, two cohorts of middle-school math teachers completed a sequence of three graduate level summer math courses: Cohort 1 (n=65) in 2009 (Course 1), 2010 (Course 2), and 2011 (Course 3); Cohort 2 (n=84) in 2011 (Course 1), 2012 (Course 2), and 2013 (Course 3). These courses

were taught at four partnering institutions of higher education, IHE, located fairly uniformly throughout a very large region of West Texas. The courses were taught by math faculty from each respective IHE, assisted by designated WTMSMP math faculty. All instructors of record had PhDs in mathematics. The focus of this paper is on the outcomes associated with teachers' completion of this mathematically intense PD experience.

Project Courses

Each course consisted of 48 contact hours taking place on the campus of each partner IHE during a two week period (three hours each Monday afternoon; six hours each Tuesday, Wednesday, Thursday; and three hours each Friday morning). Each course targeted a particular area of mathematics deemed critical for teachers of middle-school math: algebra concepts for Course 1; geometry concepts for Course 2; and probability and statistics concepts for Course 3. Each course also included a half-day workshop on an ancillary topic: Math Self-efficacy in Course 1; English Language Learners in Course 2; and Cultural Diversity in Course 3.

The textbooks for courses 1 and 2 were written by the WTMSMP Principal Investigator (PI), a mathematician with a PhD in pure math and lead author on this paper. The third course textbook was co-authored by the project PI and two other WTMSMP math faculty, one with a PhD in applied math and the other with an MS in statistics. Each textbook begins by identifying the fundamental concept in its area and then proceeds to develop the area in a mathematically logical progression. It is important to note that none of the three textbooks were aligned in any way with any preset curriculum standards or assessment measures.

The Course 1 text (*Integers and Fractions: An Investigation into the Algebraic Structure of Our Numbers*) begins with the definition of a positive integer from the view point of Bertrand Russell (Russell, 1956). The group and ring structures of the integers are derived, followed by the field structure of the rational numbers. Representations of the rational numbers, fractions and decimals, are covered in detail including the meaning of an infinitely repeating decimal "representation" of a fraction. Finally the Least Upper Bound Principle is invoked to produce the existence of irrational numbers (Harris, 2009).

The Course 2 text (*Measures of Size in 0, 1, 2, and 3 Dimensions*) begins with the size of a set of discrete points (size in 0 dimensions) being defined as the number of points in the set. A fundamental object in each subsequent higher dimension is defined via a vertical translation of the fundamental object in one lower dimension, and its size is defined to be the product of the

size of the lower dimensional object times the vertical translation distance. All the usual formulas for size of polygonal objects are derived from the sizes of the fundamental objects. The circumference of a circle stems from the definition of the number π, the area of a circle is defined in terms of the limit of the areas of regular polygons, and the volume of a sphere is gotten from the classical argument of Archimedes (Archimedes, 1912). The text ends with a discussion of fractals and fractional dimensions (Harris, 2010).

The Course 3 text (*Concepts of Probability and Statistics*) begins with the basic concept of discrete theoretical probability; namely, the probability that a point chosen randomly from a finite set of points S (sample space) is in a subset E (event) of S is the ratio $\frac{Size(E)}{Size(S)}$. After counting techniques, the concepts of random variables, probability distributions, and expected values are introduced. These concepts are then extended to continuous probability involving samples spaces in 1, 2, and 3 dimensions, leading to the standard normal density distribution. The text ends with a discussion of statistical terms and concepts often encountered in the middle grade classroom and state assessments (Harris, et al, 2011).

Project Participants

Sixty-one teachers from Cohort 1 and 85 teachers from Cohort 2 completed all three courses. In each cohort approximately 80% were women. Overall the participants reported having 0 to 40 years of experience teaching math (average a little over 9 for each cohort). In terms of mathematics background, teachers reported having taken as few as 0 college level math courses to as many as 8 (average approximately 3.5 for each cohort). Points to be stressed are

1) There is no significant difference between cohorts 1 and 2 in any of the categories reported above. (This is not surprising since they are all from the same region of Texas.)
2) There is great disparity among participants' experience teaching math.
3) There is great disparity among the undergraduate math backgrounds of the participants.

Project Results

In order to make changes if necessary, it was important to assess the project's impact on Cohort 1 during the first two years (2009 and 2010) prior to the beginning of the second Cohort 2's experience with the project in the third year (2011). Impact was investigated by determining the extent to which Cohort 1's project participation was associated with increases in key programs outcomes: mathematics conceptual knowledge (MCK), MKT, and self-efficacy for teaching mathematics?

Using widely accepted and validated MKT scales (Schilling, et al, 2007; Hill, et al, 2007;Schilling, 2007), the WTMSMP researchers were able to assess the project impact on Cohort 1 teachers' knowledge for teaching Number Concepts and Operations, Algebra, and Geometry (Harris, et al, 2011). In addition, the researchers used a locally created instrument (yet to be fully validated) to assess the impact on the teachers' MCK for geometry. All measures increased from pre-year 1 to post-year 2 with the increases in MKT for geometry and MCK being statistically significant. Since there was no alignment between the course texts and the MKT measures, the researchers were pleased to see growth in the MKT scales. There was no indication that years of experience had any relationship to participants' MKT growth. The math background of the participants did affect the MKT and MCK scores with those who had taken courses beyond college algebra scoring, on average, higher on all measures, at all instrument administrations (pre- and post-course delivery for each of the previous two summers for a total of four time points). However, the gains were parallel meaning that the value added, as a portion of initial knowledge, was the same for both groups.

The researchers believe this is a significant finding. Regardless of initial background, the rate of growth was the same for all participants. This finding is particularly interesting in light of the construct measured by the MKT measure. The MKT is not a direct measure of mathematics concepts. That is, it captures mathematics content in the context of teaching scenarios and decision making (Schilling, et al, 2007; Hill, et al, 2007;Schilling, 2007). Therefore, this finding suggests that a focus on pure mathematics can yield important changes to teachers MKT, provided the experience allows teachers' an opportunity to translate the material into meaningful classroom situations. This course feature was carefully integrated into each course. The results described in the continuing work section are consistent with this interpretation of this finding.

Using well established measures, the WTMSMP researchers also found evidence supporting participants' growth in teaching self-efficacy associated with course completion (Stevens et al., 2013a). As expected, participants reported increasingly higher levels of confidence in their ability to provide instruction and engage students at each of the four time steps. Factoring in teachers' levels of math background revealed participants who took fewer college math courses reported higher initial levels of teaching efficacy. This finding could be due to these teachers having taken more pedagogically focused courses in lieu of mathematics, although the data were not available to explore this explanation. Regardless of math background, the teaching self-

efficacy of all teachers increased, with the teaching efficacy scores of those with more math background approaching the scores of those with less math background by the end of Course 2 (Stevens et al., 2013a).

At Cohort 1's completion of WTMSMP coursework, the shape of participants' MKT growth over the three years of the project was assessed and revealed statistically significant linear growth for Algebra and Number Concepts (Stevens et al., 2013b). The teachers' growth on the Geometry MKT measure, however, was nonlinear with a large increase after Course 2 and little additional growth by the end of Course 3. Algebra and Number Concepts scores grew consistently over the three years of the project despite variation in course content. This growth is consistent with Ma's observation (Ma, 1999) that teachers' conceptual understanding of mathematics develops as they teach and interact with their students. The fact that geometry MKT scores did not grow during Course 3 could be attributed to the closer alignment of the geometry MKT measures and the content of Course 2.

Documenting Cohort 1 teachers' growth across their WTMSMP involvement was important; however, it was also important to show that participants' final MKT scores exceeded those of similar teachers who had no experience with the project. With Cohort 2 starting at the end of Cohort 1's participation, WTMWMP researchers were able to compare Cohort 1's final scores to the pretests of Cohort 2. Because the two groups of teachers reported similar levels of math background and experience teaching prior to the start of participation, this comparison was appropriate. Results indicated that although the MKT scores of Cohort 1 and Cohort 2 did not differ at the pretest, Cohort 1's final MKT scores were significantly higher than Cohort 2's pretest scores. Thus, after WTMSMP completion, participants outperformed similar teachers on measures of MKT (Stevens et al, 2013b).

Continuing Work

In addition to documenting Cohort 2 participants' growth in MKT and teaching efficacy, continuing work will also focus on what aspects of the WTMSMP project were influential in supporting all participants' growth, as well as what changes in teacher practices can be observed in middle school classrooms. Understanding what project participants perceived as most beneficial to their learning will be evaluated through analysis of the teachers' Q-sorts. "The objective in Q-methodology is to describe typical representations of different viewpoints rather than to find the proportion of individuals with specific viewpoints" (Akhtar-Danesh, Baumann,

& Cordingly, 2008). All participants were asked to sort and rank-order the aspects of the WTMSMP courses that most benefited their learning. We will use factor analysis to organize participants' perceptions into categories. This method was successfully piloted by the researchers (Stevens et al., 2009). The analysis of the Q-sort data for Cohort 1 upon completion of Course 1 revealed three approaches to learning; participants who focused on gaining competence, participants who preferred to be in control of their learning, and participants who benefited most from social learning (Stevens et al., 2013c). Cohort 1 teachers appeared to interact with Course 1 strategies and activities in different manners determined by their divergent approaches to learning. These results underscore the need to incorporate variety in course strategies and activities, and suggest the need for long term, intensive professional development activities.

To understand how participants are taking their knowledge into their classrooms, the researchers developed the Students Perceptions of Teacher Successes (SPoTS) instrument, which allows public school students the opportunity to quickly rate their teachers on key behaviors promoted by the WTMSMP project. Initial evaluation of the SPoTS yielded positive support for its usefulness in understanding teacher practices (Stevens et al., 2013d). This instrument will be used to investigate participants' ongoing use of WTMSMP content in their classrooms.

Discussion

The course structures were driven by the logical development of the math content resulting in no direct alignment with the MKT measures or the self-efficacy measures. And yet upon completion of the three courses participants had significant increases in their mathematical knowledge for teaching, math self-efficacy, and self-efficacy for teaching math. We believe there are multiple factors that contributed to these increases.

First and foremost, each course provided a deep theoretical development of the mathematics taught in the middle grades: the algebraic structure of the rational number field in Course 1; the fundamental concept of size in 0, 1, 2, and 3 dimensions in Course 2; the transition from discrete to continuous probability in Course 3. In addition the courses were taught by research mathematicians, each with a PhD in mathematics and a passion for, and appreciation of the beauty of, the mathematics being studied. Based on participant comments gathered at the final WTMSMP retreat we believe the passion and appreciation expressed by professional

mathematicians for the math encountered in their own middle-school classrooms was a source of inspiration for many of the project participants.

Another factor was the emersion effect. Each course consisted of 16 three-hour sessions completed over a two-week summer period. In each session the teachers were exposed to a theoretical development of the concepts and then worked in groups to create concrete models or demonstrations of the concepts suitable for use in their own classrooms. This produced a collaborative atmosphere in which all participants, the teachers and the IHE mathematicians, interacted as colleagues. This interaction may have contributed to a change in teachers' conceptions of mathematics and thus measurable changes in our measures.

Of the 149 teachers who began the program 132 (59 from Cohort 1 and 73 from Cohort 2) successfully completed all three courses. The majority of those who dropped did so because of changes in jobs. We believe these results demonstrate that this PD program can be of significant benefit for middle school math teachers with wide variation in teaching experience and in undergraduate mathematics backgrounds. Moreover, we believe the kind of results we are seeing are not likely attainable using the traditional professional development model: half-day, whole-day, or weekend workshops scattered throughout the year. We believe that the intense, long-term, rigorous focus on the specific mathematics content taught in the middle grades was critical to the success of our program.

References

Akhtar-Danesh, N., Baumann, A., & Cordingley, L. (2008). Q-Methodology in Nursing Research A Promising Method for the Study of Subjectivity. *Western Journal of Nursing Research*, 30(6), 759-773.

Archimedes. (1912). *Book II. On the sphere and cylinder*, New York, NY: The World of Mathematic.

Ball, D. L., Thames, M.G., &Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education 59*(5), 389-407.

Conference Board of Mathematical Sciences, 2012. *The Mathematical Education of Teachers II,* (working draft). Washington DC, www.cbmsweb.org **.**

Harris, G., *Integers and Fractions: An Investigation into the Algebraic Structure of Our Numbers*, West Texas Middle School Math Partnership, 2009. Available at http://www.wtmsmp.math.ttu.edu/ProjectActivities.htm

Harris, G., *Measures of Size in 0, 1, 2, and 3 Dimensions*, West Texas Middle School Math Partnership, 2010. Available at http://www.wtmsmp.math.ttu.edu/ProjectActivities.htm

Harris, G., Koepp, W. & Moreland, E., *Concepts of Probability and Statistics*, West Texas Middle School Math Partnership, 2011. Available at http://www.wtmsmp.math.ttu.edu/ProjectActivities.htm

Harris, G., Stevens, T., & Higgins, R. (2011). A professional development model for middle school teachers of mathematics. *International Journal of Mathematical Education in Science and Technology, 42*(7), 951-961.
Hernandez, A., Herter, R., & Wanat, S. (2008). Perceived Challenges in Working with English Learners: Meeting the Professional Development Needs of Teacher Candidates and Classroom Teachers. *The International Journal of Learning, 15*(10), 107-114.
Hill, H. C., Dean, C., & Goffney, I. M. (2007). Assessing elemental and structural validity: Data from teachers, non-teachers, and mathematicians. *Measurement, 5*(2-3), 81-92.
Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.
Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4*(2-3), 189-212.
Russell, B. (1956). *Definition of number*. New York, NY: The world of Mathematics.
Schilling, S. (2007). Generalizability and specificity of interpretive arguments: Observations inspired by the commentaries. *Measurement, 5*, 211-216.
Schilling, S. G., & Hill, H. C. (2007). Assessing measures of mathematical knowledge for teaching: A validity argument approach. *Measurement, 5*(2-3), 70-80.
Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.
Stevens, T., Aguirre-Munoz, Z., Harris, G., Higgins, R., & Liu, X. (2013a). Middle Level Mathematics Teachers' Self-Efficacy Growth through Professional Development: Differences Based on Mathematical Background. *Australian Journal of Teacher Education*, *38*(4), 9.
Stevens, T., Harris, G., Aguirre-Munoz, Z., Higgins, R. & Liu, X. (2013b) Professional Development to Facilitate Teachers' Mathematics Understanding and Associated Growth in Mathematics Knowledge for Teaching. Manuscript submitted for publication.
Stevens, T., Aguirre-Munoz, Z., Harris, G., Higgins, R., & Campos, K. (2013c). Teacher learning preferences in professional development: Self-determination implications for addressing learning needs. Manuscript in preparation.
Stevens, T., Harris, G., Aguirre-Munoz, Z., & Cobbs, L. (2009). A case study approach to increasing teachers' mathematics knowledge for teaching and strategies for building students' maths self-efficacy. *International Journal of Mathematical Education in Science and Technology*, *40*(7), 903-914.
Stevens, T., Harris, G., Liu, X., & Aguirre-Munoz, Z. (2013d). Students' ratings of teacher practices. *International Journal of Mathematical Education in Science and Technology*, *44*(7), 984-995.

Acknowledgement

The work reported in this article was supported by the National Science Foundation Math-Science Partnership program under Award No. 0831420. The opinions expressed herein are those of the authors and do not reflect the views of NSF. The authors wish to thank the referee for providing several suggestions that helped to make the paper more readable.