Final Exam
Math 1300
Fall 2011

Instructions: Solve 14 of the problems 1–16. If you solve more than 14 problems, you must clearly mark which 14 you want to be graded. For full credit, you must show complete, correct, legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, phones and PDAs are not.

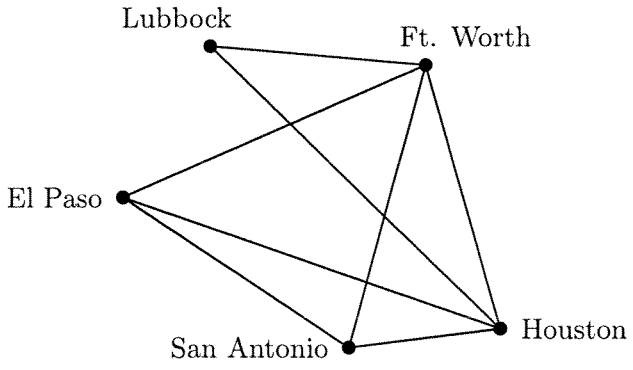
1. Write the first seven rows of Pascal's triangle and use it to answer the following questions.
   (a) In an experiment, a coin is tossed 5 times. The outcome is 2 tails and 3 heads. In how may ways can this come about?
   (b) A couple has 6 children, 3 boys and 3 girls, in how many different ways can this happen?

2. Solve each of the following equations for x (make sure to show *all* your work).
   (a) $\ln\left(\frac{x}{3}\right) + 2\ln(x) - \ln\left(\frac{1}{3}\right) - \ln(2) = \ln(1) - 2\ln\left(\frac{1}{x}\right)$
   (b) $e^{2x-1} = 4$
   (c) $e^{10} \cdot e^{-5} \cdot e^{2x-2} = e^5$

3. A political candidate decides to travel to the cities Houston, Ft. Worth, San Antonio, El Paso, and Lubbock by air. The table below shows the relevant airfares in \$.

	Ft. Worth	Houston	Lubbock	San Antonio
El Paso	402	274		289
Ft. Worth		202	179	189
Houston			238	139

   (a) Label the edges in the network below with the airfares from the table.
   (b) Use the sorted edges method to plan a cheap that starts in Lubbock and visits all five cities.
   (c) What is the cost of the trip you planned?



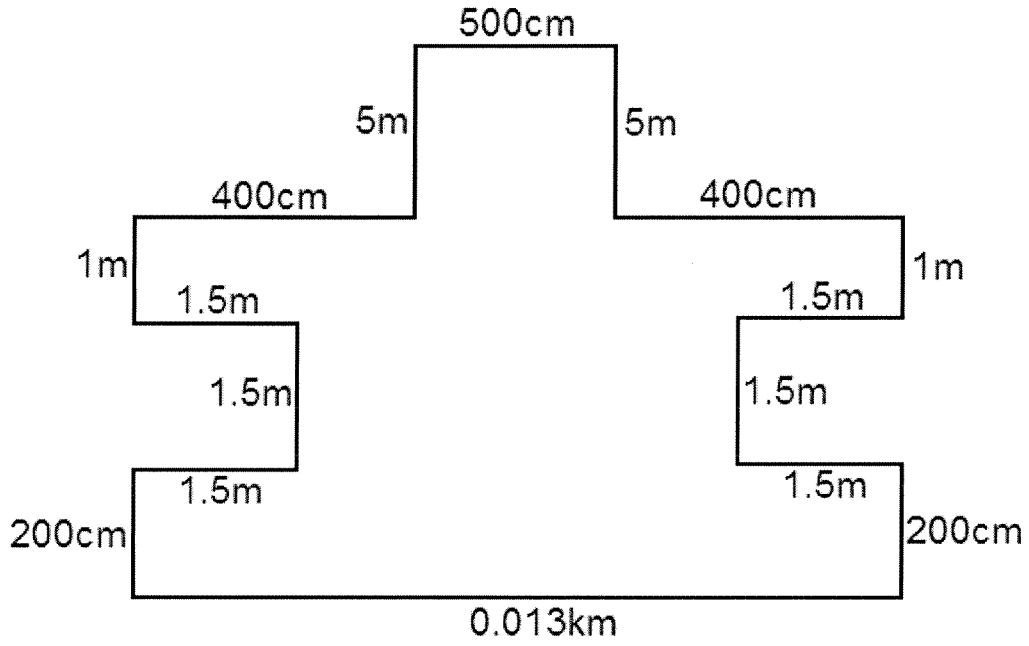
4. A charitable donation of nine used video projectors are to be distributed among six Sudanese schools. The projectors will be apportioned between the schools according to the number of students at each school; they have 144, 372, 580, 265, 153, and 286 students.

   (a) Find the total number of students and the standard divisor.

   (b) Do the apportioning according to Adam's plan.

   (c) Do the apportioning according to Jefferson's plan.

5. Consider the following figure.



   (a) Find the perimeter of the figure.

   (b) Find the area of the figure.

   (c) Imagine the figure above is the base of a 3-D object of height 4m. What is the volume of that 3-D object?

6. A survey of 50 students gave the following information:

   35 are taking a science class

   18 are taking a history class

   11 are taking both a science and a history class

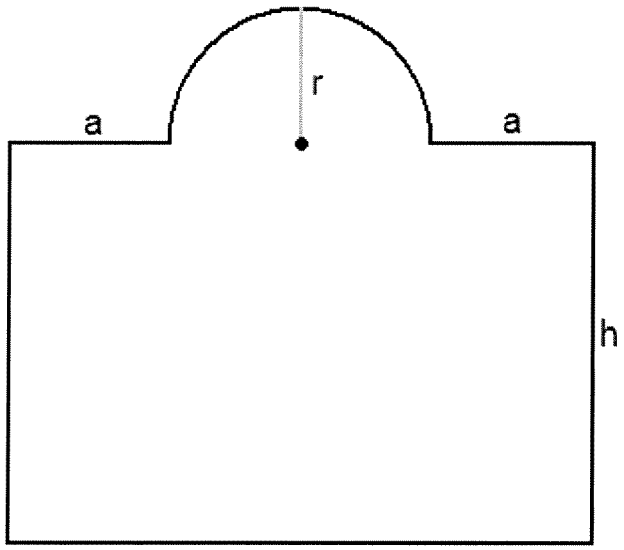
   (a) Draw a Venn diagram representing these data and fill in *every* region of the Venn diagram.

   (b) How many students are taking science but not history?

   (c) How many of the students surveyed are taking neither of these classes?

7. Say you want to save up \$2000 for a trip to Oregon in 3 years. To have that much money in 3 years, how much money should you invest now into a bank account earning 12% interest compounded monthly?

8. Consider the following figure.



   (a) Find the perimeter of the figure.

   (b) Find the area of the figure.

9. Given statements (i), (ii), and (iii), draw valid conclusions using each type of reasoning (direct, indirect, and transitive reasoning).

   (i) $p \to \sim q$

   (ii) $\sim q \to r$

   (a) (iii) $\sim q$, $\therefore$ _____ by direct reasoning

   (b) (iii) p, $\therefore$ _____ by transitive reasoning

   (c) (iii) q, $\therefore$ _____ by indirect reasoning

10. If a given population of elephants doubles in size every 5 years, find the growth rate of the heard in 1 year.

11. Set

$$U = \{1,2,3,4,5,6,7,8,9\}, \quad A = \{1,3,7,9\}, \quad \text{and} \quad B = \{1,2,4,7,8\}$$

    and perform the following set operations.

   (a) $A \cup B$.

   (b) $\overline{A} \cap B$.

   (c) $|\overline{A \cup B}|$.

12. Say you want to finance a $\$350,000$ house with a 30-year loan charging 9% interest that is compounded monthly. What will the monthly payments be?

13. In a vote between 3 candidates, the voter preferences are:

(ABC)	(ACB)	(BAC)	(BCA)	(CAB)	(CBA)
12	4	5	10	2	7

Determine the winner, if any, using 3 out of the following 5 voting methods.

(a) Majority Method

(b) Plurality Method

(c) Borda Count Method

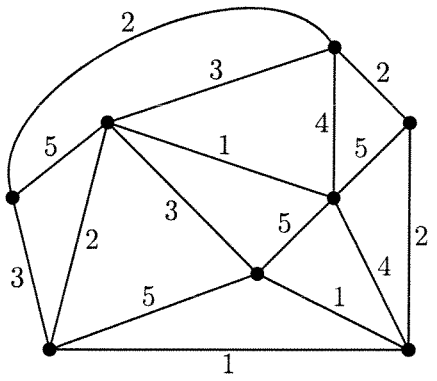
(d) Hare Method

(e) Pairwise Comparison Method

14. Is the following arguement valid? Use Euler circles to justify your answer.

Most cars have gasoline engines. Many cars have a sunroof. Therefore, some cars with a sunroof have a gasoline engine.

15. Use Kruskal's algorithm to find a minimum spanning tree for this weighted graph:



16. Prove or disprove the following:

$(\sim p \to q) \leftrightarrow (\sim q \to p)$ is a tautology.

Amortization Formula

If the amount of the loan is known (P), and you wish to find the amount of the periodic payment (m), use the formula

$$m = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}}$$

where r is the annual rate, t is the time (in years), and n is the number of payments per year.

Ordinary Annuity Formula

This is one of the most useful formulas for you to use in your personal financial planning.

The future value, A, of an annuity is found with the formula

$$A = m\left[\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}\right]$$

where r is the annual rate, m the periodic payment, t the time (in years), and n is the number of payments per year.

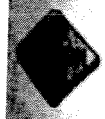
Present Value of an Annuity

If the periodic payment is known (m) and you wish to find the present value of those periodic payments, use the *present value of an annuity* formula:

$$P = m\left[\frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}}\right]$$

where P is the present value of the annuity, r is the annual interest rate, and n is the number of payments per year.

Sinking Fund Formula



Spend a few minutes with this idea; in your own words, can you explain when you would use this formula?

If the future value (A) is known, and you wish to find the amount of the periodic payment (m), use the *sinking fund formula*

$$m = \frac{A\left(\frac{r}{n}\right)}{\left(1 + \frac{r}{n}\right)^{nt} - 1}$$

where r is the annual rate, t is the time (in years), and n is the number of times per year the payments are made.

Growth/Decay Formula

This is one of the most useful formulas from mathematics.

Exponential **growth** or **decay** can be described by the equation

$$A = A_0 e^{rt}$$

where r is the annual growth/decay rate, t is the time (in years), A_0 is the amount present initially (present value), and A is the target (future) value. If r is positive, this formula models growth, and if r is negative, the formula models decay.

TABLE 17.2 Summary of Voting Methods

Method	Description
Majority Method	Each voter votes for one candidate. If the number of voters is n and n is even, then the candidate with $\frac{n}{2} + 1$ or more votes wins. If the number n is odd, then the candidate with $\frac{n+1}{2}$ or more votes wins.
Plurality Method	Each voter votes for one candidate. The candidate receiving the most votes wins.
Borda Count Method	Each voter ranks the candidates. Each last-place candidate is given 1 point, each next-to-last candidate is given 2 points, and so on. The candidate with the highest number of points wins.
Hare Method	Each voter votes for one candidate. If a candidate receives a majority of the votes, that candidate is the winner. If no candidate receives a majority, eliminate the candidate with the fewest first-place votes and repeat the process until there is a majority candidate, who wins.
Pairwise Comparison Method	Each voter ranks the candidates. Each candidate is compared to each of the other candidates. If choice A is preferred to choice B, then A receives 1 point. If B is preferred to A, then B receives one point. If the candidates tie, then each receives $\frac{1}{2}$ point. The candidate with the most points wins.
Tournament Method	This method compares the entire slate of candidates two at a time, in a predetermined order. The first and second candidates are compared, the candidate with the fewer votes is eliminated, and the winner is then compared with the third candidate. These pairwise comparisons continue until the final pairing, which selects the winner.
Approval Method	Each voter casts one vote for all the candidates that meet with his or her approval. The candidate with the most votes is declared the winner.

TABLE 17.9 Summary of Apportionment Methods

Method	Divisor	Apportionment
Adams' Plan	Round **up; raise** the standard divisor to find the modified divisor.	Round the standard quotas up. Apportion to each group its modified upper quota. It favors the smaller states.
Jefferson's Plan	Round **down; lower** the standard divisor to find the modified divisor.	Round the standard quotas down. Apportion to each group its modified lower quota. It favors the larger states.
Hamilton's Plan	Use the standard divisor. Round **down.**	Round the standard quotas down. Distribute additional seats one at a time until all items are distributed.
Webster's Plan	Use modified divisors. May round up or down.	Round by comparing with the **arithmetic mean** of the upper and lower quotas.
HH's Plan	Use modified divisors. May round up or down.	Round by comparing with the **geometric mean** of the upper and lower quotas.