Final Exam
Math 1300
Spring 2011

Instructions: Solve 14 of the problems 1–16. If you solve more than 14 problems, you must clearly mark which 14 you want to be graded. For full credit, you must show complete, correct, legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, phones and PDAs are not.

1. Perform the operations $4 + 9 \div 3 \times 2 + 2 \times 6 \div 3$.

2. Find the value of x

   - $x = \log_9 3$
   - $x = \log_4 16$
   - $x = \log_5 5^{22}$
   - $x = 2^{\log_2 4}$

3. Use Euler circles to check the validity of the following argument.

   > *All pirates are dangerous. No merchants are pirates. Therefore, no merchants are dangerous.*

   Make sure to show your Euler circles.

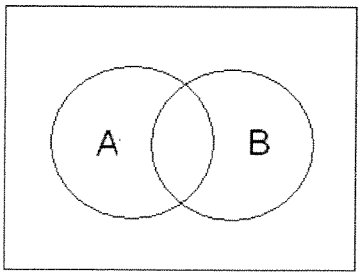
4. Set

   $$U = \{0, 1, 4, 5, 7, 8, 9, 10, 13, 15\},$$
   $$A = \{0, 4, 7, 10, 15\}, \text{ and}$$
   $$B = \{1, 5, 8, 10, 13\}$$

   and find

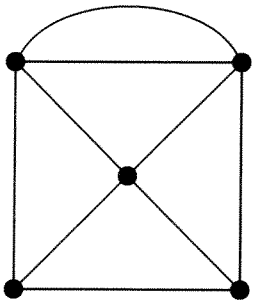
   - $\overline{A \cup B}$ and
   - $|U \times A|$.

5. Complete the following Venn diagram to show $A \cap \overline{B}$.



6. True or False? If $1 + 1 = 10$, then the Moon is made of green cheese.

7. Determine whether this network is traversable



8. Decide whether the following argument is valid or invalid.

   If I eat the ice cream, I will gain weight. I gained weight. Therefore, I ate the ice cream.

9. Find the outside surface area of a barrel that is 4 feet tall and 2 feet in diameter.

10. In an election with the following voter profiles:

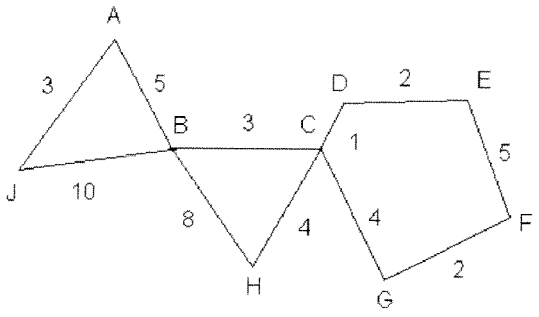
$$ABC\colon 16 \quad ACB\colon 17 \quad BAC\colon 4 \quad BCA\colon 18 \quad CAB\colon 4 \quad CBA\colon 19$$

    find the winner under the majority, plurality, hare, and Borda count methods.

11. Construct the truth table for

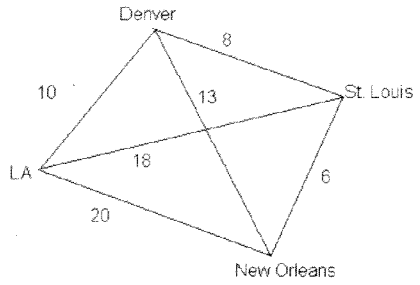
$$\sim p \vee (p \to q).$$

12. Find the minimum value of a spanning tree for this graph



Make sure to draw the minimum spanning tree and then calculate the minimum value.

13. A salesperson wants to visit each of the cities Denver, St. Louis, Los Angeles, and New Orleans. Driving distances are as shown below. Use the sorted edges method to find the shortest trip that starts in Denver and visits each of them.



14. Make the indicated conversions.

- $1\,L = ?\,ml$
- $1\,L = ?\,kl$
- $1\,km = ?\,mm$
- $1\,m^2 = ?\,cm^2$

15.
- An amount of \$5000 is invested for 7 years, calculate the simple interest earned if the investment pays 6% per year.
- Find the future value of \$5000 if the interest earned is 6% compounded semiannually for 7 years.
- Find the future value of \$5000 if the interest earned is 6% compounded monthly for 7 years.

16. A total of 20 representatives are to be divided between four cities A, B, C, and D with the following populations

$$A\text{: } 6000, \quad B\text{: } 9000, \quad C\text{: } 7400, \quad \text{and} \quad D\text{: } 2600.$$

Use Hamilton's plan to solve this apportionment problem.

MATH 1300 FORMULAS

Amortization Formula

If the amount of the loan is known (P), and you wish to find the amount of the periodic payment (m), use the formula

$$m = \frac{P\left(\frac{r}{n}\right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}}$$

where r is the annual rate, t is the time (in years), and n is the number of payments per year.

Ordinary Annuity Formula

This is one of the most useful formulas for you to use in your personal financial planning.

The future value, A, of an annuity is found with the formula

$$A = m\left[\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}\right]$$

where r is the annual rate, m the periodic payment, t the time (in years), and n the number of payments per year.

Present Value of an Annuity

If the periodic payment is known (m) and you wish to find the present value of those periodic payments, use the *present value of an annuity* formula:

$$P = m\left[\frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}}\right]$$

where P is the present value of the annuity, r is the annual interest rate, and n is the number of payments per year.

Sinking Fund Formula

Spend a few minutes with this idea; in your own words, can you explain when you would use this formula?

If the future value (A) is known, and you wish to find the amount of the periodic payment (m), use the *sinking fund formula*

$$m = \frac{A\left(\frac{r}{n}\right)}{\left(1 + \frac{r}{n}\right)^{nt} - 1}$$

where r is the annual rate, t is the time (in years), and n is the number of times per year the payments are made.

Growth/Decay Formula

This is one of the most useful formulas from mathematics.

Exponential **growth** or **decay** can be described by the equation

$$A = A_0 e^{rt}$$

where r is the annual growth/decay rate, t is the time (in years), A_0 is the amount present initially (present value), and A is the target (future) value. If r is positive, this formula models growth, and if r is negative, the formula models decay.

TABLE 17.2 Summary of Voting Methods

Method	Description
Majority Method	Each voter votes for one candidate. If the number of voters is n and n is even, then the candidate with $\frac{n}{2} + 1$ or more votes wins. If the number n is odd, then the candidate with $\frac{n+1}{2}$ or more votes wins.
Plurality Method	Each voter votes for one candidate. The candidate receiving the most votes wins.
Borda Count Method	Each voter ranks the candidates. Each last-place candidate is given 1 point, each next-to-last candidate is given 2 points, and so on. The candidate with the highest number of points wins.
Hare Method	Each voter votes for one candidate. If a candidate receives a majority of the votes, that candidate is the winner. If no candidate receives a majority, eliminate the candidate with the fewest first-place votes and repeat the process until there is a majority candidate, who wins.
Pairwise Comparison Method	Each voter ranks the candidates. Each candidate is compared to each of the other candidates. If choice A is preferred to choice B, then A receives 1 point. If B is preferred to A, then B receives one point. If the candidates tie, then each receives $\frac{1}{2}$ point. The candidate with the most points wins.
Tournament Method	This method compares the entire slate of candidates two at a time, in a predetermined order. The first and second candidates are compared, the candidate with the fewer votes is eliminated, and the winner is then compared with the third candidate. These pairwise comparisons continue until the final pairing, which selects the winner.
Approval Method	Each voter casts one vote for all the candidates that meet with his or her approval. The candidate with the most votes is declared the winner.

TABLE 17.9 Summary of Apportionment Methods

Method	Divisor	Apportionment
Adams' Plan	Round **up; raise** the standard divisor to find the modified divisor.	Round the standard quotas up. Apportion to each group its modified upper quota. It favors the smaller states.
Jefferson's Plan	Round **down; lower** the standard divisor to find the modified divisor.	Round the standard quotas down. Apportion to each group its modified lower quota. It favors the larger states.
Hamilton's Plan	Use the standard divisor. Round **down.**	Round the standard quotas down. Distribute additional seats one at a time until all items are distributed.
Webster's Plan	Use modified divisors. May round up or down.	Round by comparing with the **arithmetic mean** of the upper and lower quotas.
HH's Plan	Use modified divisors. May round up or down.	Round by comparing with the **geometric mean** of the upper and lower quotas.