

*Instructions:* Solve 13 of the problems 1–15. If you solve more than 13 problems, you must clearly mark which 13 you want to have graded. For full credit, you must show complete, correct, legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, phones and PDAs are not.

1. Given the following weighted voting system, answer the questions below.

$$[28; 10, 13, 5, 15]$$

- (a) Determine the quota and the number of voters.  
 (b) Determine the number of possible coalitions, list them all, and identify the winning coalitions.  
 (c) Find the critical voters in each winning coalition and compute the Banzhaf power index of each critical voter.
2. Consider the following preference ballot:

	8	7	4	7
1 <sup>st</sup>	A	B	C	B
2 <sup>nd</sup>	B	C	A	C
3 <sup>rd</sup>	C	D	B	A
4 <sup>th</sup>	D	A	D	D

- (a) Determine who wins the election according to the plurality with elimination method.  
 (b) Determine who wins the election according to the Borda Count method.
3. Use Euler diagrams to determine whether the following syllogism is valid or invalid.

*Some large corporations are concerned about the environment*

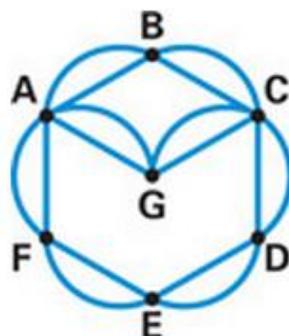
*General Motors is not a large corporation*

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*General Motors is not concerned about the environment*

4. Frank and Susanne decide to set up a college fund for their newborn grandson. They will deposit a sum of money into an account that yields 4.8% annual interest compounded monthly. What sum of money must they deposit now in order for there to be \$20,000 in the account at the end of 18 years?
5. Are the statements
- $$(p \wedge q) \rightarrow \sim q \quad \text{and} \quad q \rightarrow (\sim p \vee \sim q)$$
- logically equivalent? Justify your answer.
6. Two dice are rolled; find the probability that the total showing is less than 10.

7. The 1,218-member carpenters' union has six representatives on the state labor council, and the 70-member plumbers' union has four representatives. If either the carpenters or the plumbers can have one more representative on the council, which union is more deserving of this extra representative? Explain your reasoning
8. Consider the graph



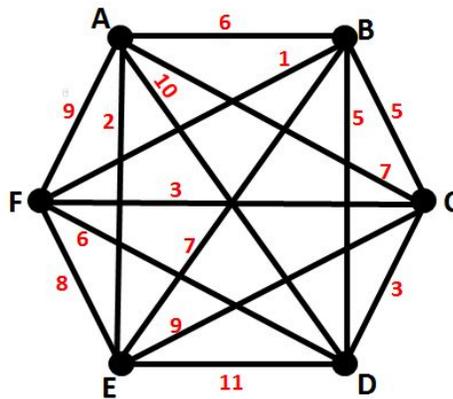
- (a) How many vertices does the graph have?
- (b) Is it connected?
- (c) Is it complete? Why?
- (d) Does it have an Euler path? Why?
9. From buying the past finals solutions at the SIAM sale, you found out that the probability of getting a question about the Borda Count method is 0.75, the probability of getting a question about the Plurality with Elimination method is 0.6, and the probability of getting a question about both is 0.45. What is the probability that you will NOT get a question about the Borda Count method OR the Plurality with Elimination method?
10. Find the mean, median, and mode for the following distribution,
- $$7, 8, 6, 5, 7, 10, 2, 7, 9, 5, 8, 8, 10, 9, 6, 5, 10, 7, 8, 9 .$$
11. Jessica, Serena, Andre, and Emily ran for president in the recent Student Government Council elections. Use the following clues to determine the order of finishing. (Who came in first, second, third, and fourth?)
- Emily has been a member of the council longer than the people who placed third and fourth in the voting.
  - Andre finished right behind Jessica and has a different major than the third-place finisher.
  - Although Serena did not win, she was happy that she did not finish last.
  - Jessica had 37 fewer votes than Emily.

12. Assume that you take out a 10-year loan for \$250,000 at an annual interest rate of 12%, and after 7 years you decide to refinance the unpaid balance of \$107,989 at a rate of 4%.
- (a) Use the table below to find the difference in your monthly payments.
- (b) What is the total interest you will have paid by the end of the 10 years?

Annual Interest Rate	Number of Years for the Loan				
	3	4	10	20	30
4%	\$29.53	\$22.58	\$10.12	\$6.06	\$4.77
5%	29.97	23.03	10.61	6.60	5.37
6%	30.42	23.49	11.10	7.16	6.00
8%	31.34	24.41	12.13	8.36	7.34
10%	32.27	25.36	13.22	9.65	8.78
12%	33.21	26.33	14.35	11.01	10.29

Monthly payments on a \$1,000 loan.

13. District *A* has a population of 7,875 people, District *B* has a population of 18,765 people, and District *C* has a population of 9,360 people. Use the Hamilton apportionment method to apportion 20 representatives between these three districts according to their populations.
14. Consider the weighted graph



Use the Nearest Neighbor algorithm to find a Hamilton circuit that begins at *A*.

15. Use the 68–95–99.7 Rule to answer the following questions about a normal distribution with a mean of 35 and a standard deviation of 5.
- (a) What percentage of values would be between 30 and 35?
- (b) What percentage of values would be below 35?
- (c) What percentage of values would be above 45?

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Method	How the Winning Candidate Is Determined
Plurality	The candidate receiving the most votes wins.
Borda count	Voters rank all candidates by assigning a set number of points to first choice, second choice, third choice, and so on; the candidate with the most points wins.
Plurality-with-elimination	Successive rounds of elections are held, with the candidate receiving the fewest votes being dropped from the ballot each time, until one candidate receives a majority of votes.
Pairwise comparison	Candidates are compared in pairs, with a point being assigned the voters' preference in each pair. (In the case of a tie, each candidate gets a half point.) After all pairs of candidates have been compared, the candidate receiving the most points wins.

**TABLE 11.8** Summary of voting methods.

### HAMILTON'S APPORTIONMENT METHOD

- Find the standard divisor for the apportionment (total population/total number of representatives).
- Find the standard quota (state's population/standard divisor) for each state and round it down to its lower quota. Assign that number of representatives to each state.
- If there are any representatives left over, assign them to states in order according to the size of the fractional parts of the states' standard quotas.

### ADAMS'S APPORTIONMENT METHOD

- Use trial and error to find a modified divisor that is larger than the standard divisor for the apportionment.
- Calculate the modified quota (state's population/modified divisor) for each state and round it up. Assign that number of representatives to each state. (Keep varying the modified divisor until the sum of these assignments is equal to the total number being apportioned.)

**THE HUNTINGTON–HILL APPORTIONMENT PRINCIPLE** If states X and Y have already been allotted  $x$  and  $y$  representatives, respectively, then state X should be given an additional representative in preference to state Y provided that

$$\frac{(\text{population of Y})^2}{y \cdot (y + 1)} < \frac{(\text{population of X})^2}{x \cdot (x + 1)}$$

Otherwise, state Y should be given the additional representative. We will often refer to a number of the form  $\frac{(\text{population of X})^2}{x \cdot (x + 1)}$  as a **Huntington–Hill number**.

**DEFINITION** In a weighted voting system, a voter's **Banzhaf power index\*** is defined as

$$\frac{\text{the number of times the voter is critical in winning coalitions}}{\text{the total number of times voters are critical in winning coalitions}}.$$

**RULE FOR COMPUTING THE PROBABILITY OF A UNION OF TWO EVENTS** If  $E$  and  $F$  are events, then

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

If  $E$  and  $F$  have no outcomes in common, they are called *mutually exclusive events*. In this case, because  $E \cap F = \emptyset$ , the preceding formula simplifies to

$$P(E \cup F) = P(E) + P(F).$$

**GENERAL RULE FOR COMPUTING  $P(F|E)$**  If  $E$  and  $F$  are events in a sample space, then  $P(F|E) = \frac{P(E \cap F)}{P(E)}$ .

**THE COMPOUND INTEREST FORMULA** Assume that an account with principal  $P$  is paying an annual interest rate  $r$  and compounding is being done  $m$  times per year. If the money remains in the account for  $n$  time periods, then the future value,  $A$ , of the account is given by the formula

$$A = P \left( 1 + \frac{r}{m} \right)^n.$$

Notice that in this formula, we have replaced  $r$  by  $\frac{r}{m}$ , which is the annual rate divided by the number of compounding periods per year, and  $t$  by  $n$ , which is the number of compounding periods.

**FORMULA FOR FINDING THE FUTURE VALUE OF AN ORDINARY**

**ANNUITY** Assume that we are making  $n$  regular payments,  $R$ , into an ordinary annuity. The interest is being compounded  $m$  times a year and deposits are made at the end of each compounding period. The future value (or amount),  $A$ , of this annuity at the end of the  $n$  periods is given by the equation

$$A = R \frac{\left( 1 + \frac{r}{m} \right)^n - 1}{\frac{r}{m}}.$$