MATH 1300
FINAL EXAM
FALL 2015

DEPARTMENT OF MATH AND STAT
TEXAS TECH UNIVERSITY

Instructions: Your answers must be entered in your Blue Examination Book; answers on the exam will not be graded. For full credit, you must show complete, correct, legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, but phones, PDAs, music players, Apple watches, and other electronic devices are not.

Part I

Solve problem 1 and make sure to explain your reasoning.

1. A person is taking a computerized driver's license test. It is multiple-choice, and for each question there are four possible answers. For every correct answer, the test taker earns one point, and for every wrong answer the test taker looses half a point.
   (a) If the test taker has no idea how to answer a particular question, is it then in her interest to choose a random answer among the four possibilities?
   (b) If the test taker can eliminate one of the four possible answers to a question, is it then in her interest to choose a random answer among the remaining three possibilities?

Part II

Solve 13 of the problems 2–16 below. If you solve more than 13 problems, then mark clearly which 13 you want graded, otherwise the first 13 answers in your Blue Examination Book will be graded.

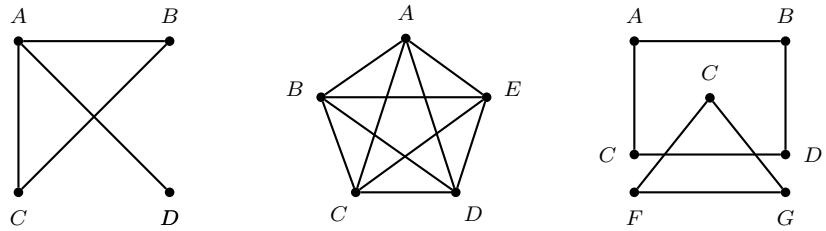
2. For each of the following statements, determine if it is true or false. If the statement is true, then give two examples to illustrate it. If the statement is false, then give a counterexample.
   (a) The equality $\frac{a}{b} - \frac{c}{d} = \frac{a-c}{b-d}$ holds.
   (b) If $a < b$, then $\frac{1}{a} < \frac{1}{b}$.
   (c) If John's monthly income was cut by 30% last summer, but this summer he was given a 30% pay raise, then is he now receiving the same income as before the original pay cut.
3. Do most Americans live to become one billion (1,000,000,000) seconds old? Explain your reasoning.
4. Determine whether the next two statements are logically equivalent and justify your answer.

$$(p \to q) \to r$$
$$p \to (q \to r)$$

5. Determine whether the following argument is valid and justify your answer.

$$\begin{array}{r} p \\ \underline{\sim q \rightarrow \sim p} \\ \therefore\ p \wedge \sim q \end{array}$$

6. For each of the graphs



(a) (b) (c)

do the following:

- If it has a Hamiltonian path, then indicate one by giving a sequence of vertices.
- If it has a Hamiltonian circuit, then indicate one by giving a sequence of vertices.

7. An emergency communications network has been set to transmit messages between several cities in Texas. The table below indicates how this network has been set up.

   (a) Draw a directed graph to model the network.

   (b) No matter which city the message originates in, can it spread throughout the entire network? Explain your reasoning.

City	*Can transmit to*
Austin	Lubbock, Houston
Beaumont	Houston, San Antonio
Dallas	Lubbock, El Paso
El Paso	Lubbock, Dallas
Houston	Beaumont, San Antonio
Lubbock	Dallas, Austin
San Antonio	Houston, Beaumont

8. The month of November has 30 days. On the first day of the month, your credit card balance is \$520. On 11/5 you charge \$289 for an airfare, on 11/15 you pay off \$500 on your account, on 11/24 you charge \$28 for gas, and on 11/28 you charge \$73 for a purchase at Amazon. Your credit card interest rate is 13.89%; use the average daily balance method to compute the finance charge on your December statement.

9. Megan wants to save up for retirement, so she sets up an ordinary annuity. If she can afford to make monthly payments of \$450 and the annuity has an annual interest rate of 3% compounded monthly, how much money will be in her account after 25 years?

10. A city is divided into five districts with the following populations:

District A:	185,000
District B:	62,000
District C:	234,000
District D:	97,000
District E:	74,000

Use Hamilton's apportionment method to assign 50 city council seats between the five districts.

11. Four candidates—W, X, Y, and Z—are running for office. Given this preference table

	Number of ballots					
Preference	15	30	18	17	10	3
First choice	W	Y	W	X	X	Z
Second choice	X	X	Z	Y	Z	W
Third choice	Z	Z	Y	Z	W	X
Fourth choice	Y	W	X	W	Y	Y

determine who wins using the Borda Count Method. Does the same candidate win when using the Plurality Method?

12. At TTU, a board represents residents of Knapp Hall, Horn Hall, and Bledsoe Hall. The board has 7 members. Using the following table of Huntington–Hill numbers, list the order in which the board will be appointed.

Current Number of Representatives	(K)napp	(H)orn	(B)ledsoe
1	272.5	112.5	350.0
2	108.5	75.5	199.2
3	52.2	45.0	90.0
4	25	10	46.5

13. In the voting system

$$[10 : 8, 4, 2, 1]$$

the weights represent, in order, voters A, B, C, and D.

(a) Find all the winning coalitions; determine the critical voters in each winning coalition.

(b) Compute the Banzhaf power index for each voter.

14. This semester there are 29,237 undergraduate students enrolled at TTU and 7,017 of them take Math classes. There are 530 students enrolled in Math 1300 Contemporary Mathematics and 1,233 students enrolled in Math 1320 College Algebra; among these there are 65 who are enrolled in both classes. Find

(a) The probability that a randomly chosen undergraduate student is taking a Math class.

(b) The probability that a student who takes a Math class takes Math 1320.

(c) The probability that a student who takes a Math 1320 is also taking Math 1300.

15. The normal monthly precipitation (in inches) for August is listed for 20 different U.S. cities.

3.9	4.2	3.6	1.0	3.7
0.4	7.0	1.5	4.2	1.6
3.7	2.4	2.7	4.1	3.4
2.0	3.4	3.6	2.2	3.5

(a) Find the mean and median of the precipitation

(b) Construct a box-and-whisker plot for the given data

16. Assume that a normal distribution has a mean of 24 and a standard deviation of 7. What percentage of the data values in the distribution do we expect to fall between 17 and 24?

THE UNPAID BALANCE METHOD FOR COMPUTING THE FINANCE CHARGE ON A CREDIT CARD LOAN This method also uses the simple interest formula $I = Prt$; however,

P = previous month's balance + finance charge + purchases made − returns − payments.

The variable r is the annual interest rate, and $t = \frac{1}{12}$.

THE AVERAGE DAILY BALANCE METHOD FOR COMPUTING THE FINANCE CHARGE ON A CREDIT CARD LOAN

1. Add the outstanding balance for your account for each day of the month.
2. Divide the total in step 1 by the number of days in the month to find the average daily balance.
3. To find the finance charge, use the formula $I = Prt$, where P is the average daily balance found in step 2, r is the annual interest rate, and t is the number of days in the month divided by 365.

FORMULA FOR FINDING THE FUTURE VALUE OF AN ORDINARY ANNUITY Assume that we are making n regular payments, R, into an ordinary annuity. The interest is being compounded m times a year and deposits are made at the end of each compounding period. The future value (or amount), A, of this annuity at the end of the n periods is given by the equation

$$A = R\frac{\left(1 + \frac{r}{m}\right)^n - 1}{\frac{r}{m}}.$$

FINDING THE PRESENT VALUE OF AN ANNUITY Assume that you are making m periodic payments per year for n total payments into an annuity that pays an annual interest rate of r. Also assume that each of your payments is R. Then to find the present value of your annuity, solve for P in the equation

$$P\left(1 + \frac{r}{m}\right)^n = R\left(\frac{\left(1 + \frac{r}{m}\right)^n - 1}{\frac{r}{m}}\right).$$

GENERAL RULE FOR COMPUTING $P(F|E)$ If E and F are events in a sample space, then $P(F|E) = \frac{P(E \cap F)}{P(E)}$.

RULE FOR COMPUTING THE PROBABILITY OF A UNION OF TWO EVENTS If E and F are events, then

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

If E and F have no outcomes in common, they are called *mutually exclusive events.* In this case, because $E \cap F = \varnothing$, the preceding formula simplifies to

$$P(E \cup F) = P(E) + P(F).$$

DEFINITION Assume that an experiment has outcomes numbered 1 to n with probabilities $P_1, P_2, P_3, \ldots, P_n$. Assume that each outcome has a numerical value associated with it and these are labeled $V_1, V_2, V_3, \ldots, V_n$. The **expected value** of the experiment is

$$(P_1 \cdot V_1) + (P_2 \cdot V_2) + (P_3 \cdot V_3) + \cdots + (P_n \cdot V_n).$$

DEFINITION In a weighted voting system, a voter's **Banzhaf power index*** is defined as

$$\frac{\text{the number of times the voter is critical in winning coalitions}}{\text{the total number of times voters are critical in winning coalitions}}.$$

HAMILTON'S APPORTIONMENT METHOD

a) Find the standard divisor for the apportionment (total population/total number of representatives).

b) Find the standard quota (state's population/standard divisor) for each state and round it down to its lower quota. Assign that number of representatives to each state.

c) If there are any representatives left over, assign them to states in order according to the size of the fractional parts of the states' standard quotas.

THE HUNTINGTON–HILL APPORTIONMENT PRINCIPLE If states X and Y have already been allotted x and y representatives, respectively, then state X should be given an additional representative in preference to state Y provided that

$$\frac{(\text{population of Y})^2}{y \cdot (y+1)} < \frac{(\text{population of X})^2}{x \cdot (x+1)}$$

Otherwise, state Y should be given the additional representative. We will often refer to a number of the form $\dfrac{(\text{population of X})^2}{x \cdot (x+1)}$ as a **Huntington–Hill number.**

z	A	z	A	z	A	z	A	z	A	z	A
.00	.000	.56	.212	1.12	.369	1.68	.454	2.24	.488	2.80	.497
.01	.004	.57	.216	1.13	.371	1.69	.455	2.25	.488	2.81	.498
.02	.008	.58	.219	1.14	.373	1.70	.455	2.26	.488	2.82	.498
.03	.012	.59	.222	1.15	.375	1.71	.456	2.27	.488	2.83	.498
.04	.016	.60	.226	1.16	.377	1.72	.457	2.28	.489	2.84	.498
.05	.020	.61	.229	1.17	.379	1.73	.458	2.29	.489	2.85	.498
.06	.024	.62	.232	1.18	.381	1.74	.459	2.30	.489	2.86	.498
.07	.028	.63	.236	1.19	.383	1.75	.460	2.31	.490	2.87	.498
.08	.032	.64	.239	1.20	.385	1.76	.461	2.32	.490	2.88	.498
.09	.036	.65	.242	1.21	.387	1.77	.462	2.33	.490	2.89	.498
.10	.040	.66	.245	1.22	.389	1.78	.463	2.34	.490	2.90	.498
.11	.044	.67	.249	1.23	.391	1.79	.463	2.35	.491	2.91	.498
.12	.048	.68	.252	1.24	.393	1.80	.464	2.36	.491	2.92	.498
.13	.052	.69	.255	1.25	.394	1.81	.465	2.37	.491	2.93	.498
.14	.056	.70	.258	1.26	.396	1.82	.466	2.38	.491	2.94	.498
.15	.060	.71	.261	1.27	.398	1.83	.466	2.39	.492	2.95	.498
.16	.064	.72	.264	1.28	.400	1.84	.467	2.40	.492	2.96	.499
.17	.068	.73	.267	1.29	.402	1.85	.468	2.41	.492	2.97	.499
.18	.071	.74	.270	1.30	.403	1.86	.469	2.42	.492	2.98	.499
.19	.075	.75	.273	1.31	.405	1.87	.469	2.43	.493	2.99	.499
.20	.079	.76	.276	1.32	.407	1.88	.470	2.44	.493	3.00	.499
.21	.083	.77	.279	1.33	.408	1.89	.471	2.45	.493	3.01	.499
.22	.087	.78	.282	1.34	.410	1.90	.471	2.46	.493	3.02	.499
.23	.091	.79	.285	1.35	.412	1.91	.472	2.47	.493	3.03	.499
.24	.095	.80	.288	1.36	.413	1.92	.473	2.48	.493	3.04	.499
.25	.099	.81	.291	1.37	.415	1.93	.473	2.49	.494	3.05	.499
.26	.103	.82	.294	1.38	.416	1.94	.474	2.50	.494	3.06	.499
.27	.106	.83	.297	1.39	.418	1.95	.474	2.51	.494	3.07	.499
.28	.110	.84	.300	1.40	.419	1.96	.475	2.52	.494	3.08	.499
.29	.114	.85	.302	1.41	.421	1.97	.476	2.53	.494	3.09	.499
.30	.118	.86	.305	1.42	.422	1.98	.476	2.54	.495	3.10	.499
.31	.122	.87	.308	1.43	.424	1.99	.477	2.55	.495	3.11	.499
.32	.126	.88	.311	1.44	.425	2.00	.477	2.56	.495	3.12	.499
.33	.129	.89	.313	1.45	.427	2.01	.478	2.57	.495	3.13	.499
.34	.133	.90	.316	1.46	.428	2.02	.478	2.58	.495	3.14	.499
.35	.137	.91	.319	1.47	.429	2.03	.479	2.59	.495	3.15	.499
.36	.141	.92	.321	1.48	.431	2.04	.479	2.60	.495	3.16	.499
.37	.144	.93	.324	1.49	.432	2.05	.480	2.61	.496	3.17	.499
.38	.148	.94	.326	1.50	.433	2.06	.480	2.62	.496	3.18	.499
.39	.152	.95	.329	1.51	.435	2.07	.481	2.63	.496	3.19	.499
.40	.155	.96	.332	1.52	.436	2.08	.481	2.64	.496	3.20	.499
.41	.159	.97	.334	1.53	.437	2.09	.482	2.65	.496	3.21	.499
.42	.163	.98	.337	1.54	.438	2.10	.482	2.66	.496	3.22	.499
.43	.166	.99	.339	1.55	.439	2.11	.483	2.67	.496	3.23	.499
.44	.170	1.00	.341	1.56	.441	2.12	.483	2.68	.496	3.24	.499
.45	.174	1.01	.344	1.57	.442	2.13	.483	2.69	.496	3.25	.499
.46	.177	1.02	.346	1.58	.443	2.14	.484	2.70	.497	3.26	.499
.47	.181	1.03	.349	1.59	.444	2.15	.484	2.71	.497	3.27	.500
.48	.184	1.04	.351	1.60	.445	2.16	.485	2.72	.497	3.28	.500
.49	.188	1.05	.353	1.61	.446	2.17	.485	2.73	.497	3.29	.500
.50	.192	1.06	.355	1.62	.447	2.18	.485	2.74	.497	3.30	.500
.51	.195	1.07	.358	1.63	.449	2.19	.486	2.75	.497	3.31	.500
.52	.199	1.08	.360	1.64	.450	2.20	.486	2.76	.497	3.32	.500
.53	.202	1.09	.362	1.65	.451	2.21	.487	2.77	.497	3.33	.500
.54	.205	1.10	.364	1.66	.452	2.22	.487	2.78	.497		
.55	.209	1.11	.367	1.67	.453	2.23	.487	2.79	.497		