MATH 1300
FINAL EXAM
SPRING 2015

DEPARTMENT OF MATH AND STAT
TEXAS TECH UNIVERSITY

Instructions: For full credit, you must show complete, correct, legible work. All work and answers must be recorded in your bluebook. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, phones and similar devices are not.

Part 1: Answer the first question. To earn credit on this question, you must show your work and write a sentence explaining how you got your solution.

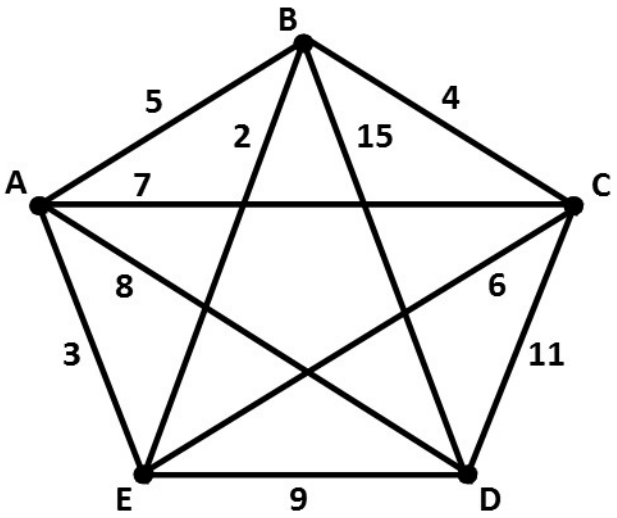
1. A mother is 20 years older than her son. Six years from now, the mother will be three times as old as her son. Based on this information, what is the current age of the son?

Part 2: Solve twelve of the problems numbered 2-15. State in your bluebook which two questions that you do not want graded. If you solve more than twelve of these problems and do not mark which two problems you do not want graded, the first twelve problems listed in your bluebook will be graded.

2. Construct a truth table for the following statement:

$$[\sim (p \to q)] \vee (p \wedge \sim q)$$

3. For the graph below, use the nearest neighbor algorithm to find a Hamilton circuit that begins at vertex A. Describe the circuit by listing its vertices and state the weight of the circuit.



4. A bank gives you two options to choose from for your investments:

   - Option A: 4% annual interest rate compounded yearly.
   - Option B: 3.95% annual interest rate compounded quarterly.

   Decide which option is the better investment at the end of two years. Justify your answer.

5. A country with two states, A and B, has 75 seats in the legislature. The population of state A (in thousands) is 3,184 while the population of state B (in thousands) is 8,475.

   (a) Use Hamilton's method to assign the seats between the two states.

   (b) Now suppose a third state, state C, with a population of 330 (in thousands), is added to the country. To account for this, the total number of seats is increased by 2. Reapportion the 77 seats between the three states using Hamilton's method.

   (c) Does the new states paradox occur?

6. Consider the following voting preference table:

	5	5	3	3
1st	A	C	B	C
2nd	B	B	C	B
3rd	D	D	D	A
4th	C	A	A	D

   (a) Who wins the election according to the plurality with elimination method?

   (b) Who wins the election according to the Borda count method?

7. A bag contains 24 red marbles, 13 blue marbles, 18 green marbles, and 5 yellow marbles.

   (a) If you randomly draw one marble from the bag, what is the probability that it is blue or green?

   (b) If you randomly draw two marbles from the bag without replacement, what is the probability that the second marble drawn is red, assuming that the first marble you drew was also red?

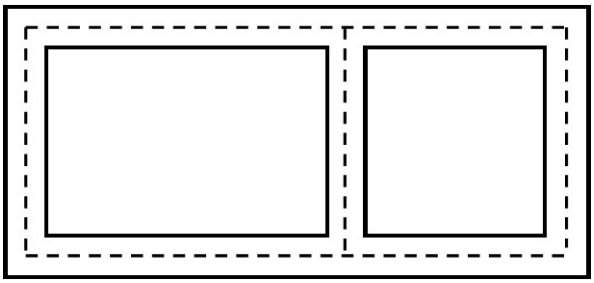
8. Consider a bag of chocolate chips. Assume that the number of chocolate chips in each bag is normally distributed with a mean of 350 chocolate chips and a standatd deviation of 8 chocolate chips. What percent of bags would we expect to have less than 360 chocolate chips?

9. Use Euler diagrams to determine whether the following syllogism is valid or invalid.

   Some rock bands play good music.
   All bands from Lubbock play good music.

   Therefore, some bands from Lubbock are rock bands.

10. Consider the following street map. Suppose that you want to travel along every street in an efficient way so that there are a minimal number of streets that you travel over more than once.



   (a) Construct a graph that models the street map. Label the vertices A-F, starting in the top left corner, and proceeding alphabetically in a clockwise manner.

   (b) Eulerize the graph, then find an Euler circuit starting at vertex A. What edge or edges must be added to Eulerize the graph? Describe the Euler circuit by listing its vertices.

11. Use the unpaid balance method to find the finance charge on the credit card account for the month of May (31 days). The previous month's balance was \$950, the annual interest rate is 21%, and the following transactions occured:

Date	**Transaction**
May 3	Made a payment of \$370 towards account balance.
May 12	Charged \$106 to card for a rental car.
May 21	Charged \$95 to card to buy museum tickets.
May 29	Returned a camera that cost \$69.

12. Subdivision A, with 1,600 residents, has 11 representatives in City Hall; Subdivision B, with 2,650 residents, has 17 representatives in City Hall; and Subdivision C, with 950 residents, has 7 representatives in City Hall. Use the Huntington-Hill apportionment principle to decide which subdivision is most deserving of getting one additional representative.

13. Consider the weighted voting system

$$[\,17 : 8, 4, 9, 3\,]$$

where the weights represent voters A, B, C, and D respectively.

(a) State the quota, state the number of voters, and determine the total number of coalitions possible within this weighted voting system.

(b) List all possible coalitions, identify which of the coalitions are winning, and determine which voters are critical in each winning coalition.

(c) Compute the Banzhaf power index for each voter.

14. One thousand raffle tickets are sold for \$1 each. There is one grand prize of \$400, two second prizes of \$50, and five third prizes of \$10.

(a) What is the expected value of buying one raffle ticket?

(b) Is the game fair? If not, determine the raffle ticket price that would make the game fair.

15. Consider the following distribution:

$$21, 24, 28, 28, 29, 35, 36, 40, 56, 70, 95$$

(a) Find the mean.

(b) Find the five-number summary.

(c) Construct the corresponding box-and-whisker plot.

THE COMPOUND INTEREST FORMULA Assume that an account with principal P is paying an annual interest rate r and compounding is being done m times per year. If the money remains in the account for n time periods, then the future value, A, of the account is given by the formula

$$A = P\left(1 + \frac{r}{m}\right)^n.$$

Notice that in this formula, we have replaced r by $\frac{r}{m}$, which is the annual rate divided by the number of compounding periods per year, and t by n, which is the number of compounding periods.

THE UNPAID BALANCE METHOD FOR COMPUTING THE FINANCE CHARGE ON A CREDIT CARD LOAN This method also uses the simple interest formula $I = Prt$; however,

P = previous month's balance + finance charge + purchases made − returns − payments.

The variable r is the annual interest rate, and $t = \frac{1}{12}$.

HAMILTON'S APPORTIONMENT METHOD

a) Find the standard divisor for the apportionment (total population/total number of representatives).

b) Find the standard quota (state's population/standard divisor) for each state and round it down to its lower quota. Assign that number of representatives to each state.

c) If there are any representatives left over, assign them to states in order according to the size of the fractional parts of the states' standard quotas.

THE HUNTINGTON–HILL APPORTIONMENT PRINCIPLE If states X and Y have already been allotted x and y representatives, respectively, then state X should be given an additional representative in preference to state Y provided that

$$\frac{(\text{population of Y})^2}{y \cdot (y+1)} < \frac{(\text{population of X})^2}{x \cdot (x+1)}$$

Otherwise, state Y should be given the additional representative. We will often refer to a number of the form $\frac{(\text{population of X})^2}{x \cdot (x+1)}$ as a **Huntington–Hill number.**

Method	How the Winning Candidate Is Determined
Plurality	The candidate receiving the most votes wins.
Borda count	Voters rank all candidates by assigning a set number of points to first choice, second choice, third choice, and so on; the candidate with the most points wins.
Plurality-with-elimination	Successive rounds of elections are held, with the candidate receiving the fewest votes being dropped from the ballot each time, until one candidate receives a majority of votes.
Pairwise comparison	Candidates are compared in pairs, with a point being assigned the voters' preference in each pair. (In the case of a tie, each candidate gets a half point.) After all pairs of candidates have been compared, the candidate receiving the most points wins.

RULE FOR COMPUTING THE PROBABILITY OF A UNION OF TWO EVENTS If E and F are events, then

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

GENERAL RULE FOR COMPUTING $P(F|E)$ If E and F are events in a sample space, then $P(F|E) = \dfrac{P(E \cap F)}{P(E)}$.

FORMULA FOR CONVERTING RAW SCORES TO z-SCORES Assume a normal distribution has a mean of μ and a standard deviation of σ. We use the equation

$$z = \frac{x - \mu}{\sigma}$$

to convert a value x in the nonstandard distribution to a z-score.

z	A	z	A	z	A	z	A	z	A	z	A
.00	.000	.56	.212	1.12	.369	1.68	.454	2.24	.488	2.80	.497
.01	.004	.57	.216	1.13	.371	1.69	.455	2.25	.488	2.81	.498
.02	.008	.58	.219	1.14	.373	1.70	.455	2.26	.488	2.82	.498
.03	.012	.59	.222	1.15	.375	1.71	.456	2.27	.488	2.83	.498
.04	.016	.60	.226	1.16	.377	1.72	.457	2.28	.489	2.84	.498
.05	.020	.61	.229	1.17	.379	1.73	.458	2.29	.489	2.85	.498
.06	.024	.62	.232	1.18	.381	1.74	.459	2.30	.489	2.86	.498
.07	.028	.63	.236	1.19	.383	1.75	.460	2.31	.490	2.87	.498
.08	.032	.64	.239	1.20	.385	1.76	.461	2.32	.490	2.88	.498
.09	.036	.65	.242	1.21	.387	1.77	.462	2.33	.490	2.89	.498
.10	.040	.66	.245	1.22	.389	1.78	.463	2.34	.490	2.90	.498
.11	.044	.67	.249	1.23	.391	1.79	.463	2.35	.491	2.91	.498
.12	.048	.68	.252	1.24	.393	1.80	.464	2.36	.491	2.92	.498
.13	.052	.69	.255	1.25	.394	1.81	.465	2.37	.491	2.93	.498
.14	.056	.70	.258	1.26	.396	1.82	.466	2.38	.491	2.94	.498
.15	.060	.71	.261	1.27	.398	1.83	.466	2.39	.492	2.95	.498
.16	.064	.72	.264	1.28	.400	1.84	.467	2.40	.492	2.96	.499
.17	.068	.73	.267	1.29	.402	1.85	.468	2.41	.492	2.97	.499
.18	.071	.74	.270	1.30	.403	1.86	.469	2.42	.492	2.98	.499
.19	.075	.75	.273	1.31	.405	1.87	.469	2.43	.493	2.99	.499
.20	.079	.76	.276	1.32	.407	1.88	.470	2.44	.493	3.00	.499
.21	.083	.77	.279	1.33	.408	1.89	.471	2.45	.493	3.01	.499
.22	.087	.78	.282	1.34	.410	1.90	.471	2.46	.493	3.02	.499
.23	.091	.79	.285	1.35	.412	1.91	.472	2.47	.493	3.03	.499
.24	.095	.80	.288	1.36	.413	1.92	.473	2.48	.493	3.04	.499
.25	.099	.81	.291	1.37	.415	1.93	.473	2.49	.494	3.05	.499
.26	.103	.82	.294	1.38	.416	1.94	.474	2.50	.494	3.06	.499
.27	.106	.83	.297	1.39	.418	1.95	.474	2.51	.494	3.07	.499
.28	.110	.84	.300	1.40	.419	1.96	.475	2.52	.494	3.08	.499
.29	.114	.85	.302	1.41	.421	1.97	.476	2.53	.494	3.09	.499
.30	.118	.86	.305	1.42	.422	1.98	.476	2.54	.495	3.10	.499
.31	.122	.87	.308	1.43	.424	1.99	.477	2.55	.495	3.11	.499
.32	.126	.88	.311	1.44	.425	2.00	.477	2.56	.495	3.12	.499
.33	.129	.89	.313	1.45	.427	2.01	.478	2.57	.495	3.13	.499
.34	.133	.90	.316	1.46	.428	2.02	.478	2.58	.495	3.14	.499
.35	.137	.91	.319	1.47	.429	2.03	.479	2.59	.495	3.15	.499
.36	.141	.92	.321	1.48	.431	2.04	.479	2.60	.495	3.16	.499
.37	.144	.93	.324	1.49	.432	2.05	.480	2.61	.496	3.17	.499
.38	.148	.94	.326	1.50	.433	2.06	.480	2.62	.496	3.18	.499
.39	.152	.95	.329	1.51	.435	2.07	.481	2.63	.496	3.19	.499
.40	.155	.96	.332	1.52	.436	2.08	.481	2.64	.496	3.20	.499
.41	.159	.97	.334	1.53	.437	2.09	.482	2.65	.496	3.21	.499
.42	.163	.98	.337	1.54	.438	2.10	.482	2.66	.496	3.22	.499
.43	.166	.99	.339	1.55	.439	2.11	.483	2.67	.496	3.23	.499
.44	.170	1.00	.341	1.56	.441	2.12	.483	2.68	.496	3.24	.499
.45	.174	1.01	.344	1.57	.442	2.13	.483	2.69	.496	3.25	.499
.46	.177	1.02	.346	1.58	.443	2.14	.484	2.70	.497	3.26	.499
.47	.181	1.03	.349	1.59	.444	2.15	.484	2.71	.497	3.27	.500
.48	.184	1.04	.351	1.60	.445	2.16	.485	2.72	.497	3.28	.500
.49	.188	1.05	.353	1.61	.446	2.17	.485	2.73	.497	3.29	.500
.50	.192	1.06	.355	1.62	.447	2.18	.485	2.74	.497	3.30	.500
.51	.195	1.07	.358	1.63	.449	2.19	.486	2.75	.497	3.31	.500
.52	.199	1.08	.360	1.64	.450	2.20	.486	2.76	.497	3.32	.500
.53	.202	1.09	.362	1.65	.451	2.21	.487	2.77	.497	3.33	.500
.54	.205	1.10	.364	1.66	.452	2.22	.487	2.78	.497		
.55	.209	1.11	.367	1.67	.453	2.23	.487	2.79	.497		

TABLE 14.15 Standard normal distribution.