## Instructions

Your answers must be entered in your Examination Blue Book; answers on the exam will not be graded. For full credit, you must show complete, correct, legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, but phones, PDAs, music players, Apple watches, and other electronic devices are not.

Solve problem 1, and solve any 13 of the problems 2–16; they are weighted equally. If you solve more than 13 of the problems 2–16, then mark clearly which ones you want graded, otherwise the first 13 answers in your Examination Blue Book will be graded.

## Part I

*Solve problem 1 and make sure to explain your reasoning.*

1. A box contains chocolate chip cookies, snickerdoodle cookies, and oatmeal raisin cookies. There are a total of 50 cookies in the box. If there are twice as many chocolate chip cookies as there are snickerdoodle cookies, and two more oatmeal raisin cookies than snickerdoodle cookies, how many of each type of cookie are in the box?

## Part II

*Solve 13 of the problems 2–16 below. If you solve more than 13 problems, then mark clearly which ones you want graded, otherwise the first 13 answers in your Examination Blue Book will be graded.*

2. A community college has five schools with the following number of students:

| | |
|---|---|
| Business | 1,835 |
| Music | 281 |
| Nursing | 1,282 |
| Social Work | 694 |
| Technology | 1,004 |

   Use Hamilton's method to apportion the 21 seats on the Student Government Board.

3. A university has an enrollment of 28,422 students. Of this total number, 23,107 are undergraduate students and the rest are graduate students. The school of Engineering has 4,349 undergraduate students and 673 graduate students.

   A student is chosen at random among the 28,422 enrolled students.

   (a) Find the probability that it is a graduate student.

   (b) Find the probability that it is an engineering student.

   (c) Given that it is an engineering student, find the probability that it is a graduate student.

4. Listed below are the contenders for the Democratic and Republican nominations for the 2016 presidential election. The numbers in parentheses are their current age; give the Five-Number summary of this data set and illustrate it with a box-and-whisker plot.

   Bush (63), Carson (64), Christie (53), Clinton (68), Cruz (44), Fiorina (61), Gilmore (66), Graham (60), Huckabee (60), Jindal (44), Kasich (63), O'Malley (53), Pataki (70), Rubio (44), Ryan (53), Santorum (57), Sanders (74), and Trump (69).

5. Use inductive reasoning to predict the next term in the following sequences:

   (a) 95, 88, 81, 74, 67, __

   (b) 8, 4, 2, 1, $\frac{1}{2}$, __

   (c) 1, 1, 2, 3, 5, 8, 13, 21, __

6. Consider the weighted voting system

$$[\,24 : 15, 12, 10\,]$$

where the weights represent, in order, voters $A$, $B$, and $C$.

   (a) List all possible coalitions and their weights, identify the winning coalitions, and determine which voters are critical in each winning coalition.

   (b) Compute the Banzhaf power index for each voter.

7. In a game of chance, you roll a standard 6-sided die. If you roll a 1, you win \$10. If you roll a 2 or 3, you win \$1. If you roll a 4, 5, or 6, you lose \$5. What is the expected value of playing this game one time?

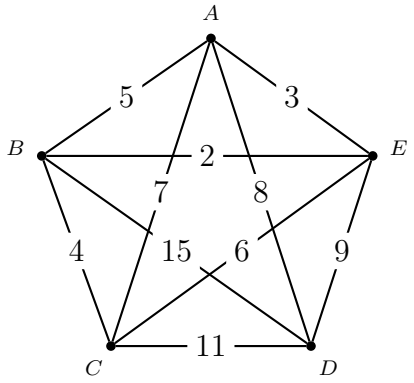
8. Assume that the time customers spend waiting in the drive-thru line to get their food at a certain fast food restaurant is normally distributed with a mean wait time of 6 minutes and a standard deviation of 1.5 minutes. Use the 68–95–99.7 Rule to determine the percentage of customers that would wait:

   (a) Between 3 and 6 minutes.

   (b) Less than 6 minutes.

   (c) More than 7.5 minutes.

9. Consider the following graph with weighted edges




   Find a low weight Hamilton circuit using

   (a) The Nearest Neighbor Algorithm starting at vertex B.

   (b) The Best Edge Algorithm.

10. State A has a population of 6.8 million, state B has a population of 14.2 million, and state C has a population of 12.4 million. Use the Huntington–Hill method to assign 8 representatives to the three states; start by giving each state one representative.

11. Construct a truth table for the statement

$$(\sim p \vee \sim q) \longrightarrow (q \wedge p)$$

12. (a) Represent the map below by a graph, and determine if the graph is connected.
    (b) Find the number of odd vertices.
    (c) Can the graph be traced? Explain why or why not.
    (d) If possible, use Fleury's algorithm to find an Euler circuit.


13. The Jones family is buying a new house at the price of $165,000; they will finance it with a 20 year mortgage that has an interest rate of 8%.
    (a) Assuming that the family can make a $39,000 down payment, what will their monthly mortgage payment be? You may use the table on the formula sheet for this computation.
    (b) If the family could increase the down payment by $10,000, how much would their monthly mortgage payment be? You may use the table on the formula sheet for this computation.
    (c) In total, how much money can the family save by making the larger downpayment?

14. Use Euler diagrams to determine whether the following syllogism is valid or invalid.

    *Some teachers believe in corporal punishment.*
    *All who believe in corporal punishment are cruel.*

    ---

    $\therefore$ *Some teachers are cruel.*

15. James and Anne are establishing a fund for their daughter's college education. What lump sum must they deposit in an account that gives 6% annual interest compounded monthly in order for the account to hold $65,000 after 10 years?

16. A city council gave a questionnaire asking the citizens to rank their priorities for next year's budget. The choices were (P)olice, (R)oad, (S)chool, and (T)rash removal. A total of 118 citizens responded, and their priorities were:

| | *Number of respondents* | | | | | |
|---|---|---|---|---|---|---|
| *Preference* | 30 | 30 | 20 | 14 | 20 | 4 |
| 1st | P | T | P | R | R | S |
| 2nd | R | R | R | T | S | P |
| 3rd | S | S | T | S | P | R |
| 4th | T | P | S | P | T | T |

    Which of the four priorities wins under the Plurality-with-Elimination method?

**HAMILTON'S APPORTIONMENT METHOD**

a) Find the standard divisor for the apportionment (total population/total number of representatives).

b) Find the standard quota (state's population/standard divisor) for each state and round it down to its lower quota. Assign that number of representatives to each state.

c) If there are any representatives left over, assign them to states in order according to the size of the fractional parts of the states' standard quotas.

| Method | How the Winning Candidate Is Determined |
|---|---|
| Plurality | The candidate receiving the most votes wins. |
| Borda count | Voters rank all candidates by assigning a set number of points to first choice, second choice, third choice, and so on; the candidate with the most points wins. |
| Plurality-with-elimination | Successive rounds of elections are held, with the candidate receiving the fewest votes being dropped from the ballot each time, until one candidate receives a majority of votes. |
| Pairwise comparison | Candidates are compared in pairs, with a point being assigned the voters' preference in each pair. (In the case of a tie, each candidate gets a half point.) After all pairs of candidates have been compared, the candidate receiving the most points wins. |

| | Number of Years for the Loan | | | | |
|---|---|---|---|---|---|
| Annual Interest Rate | 3 | 4 | 10 | 20 | 30 |
| 4% | $29.53 | $22.58 | $10.12 | $6.06 | $4.77 |
| 5% | 29.97 | 23.03 | 10.61 | 6.60 | 5.37 |
| 6% | 30.42 | 23.49 | 11.10 | 7.16 | 6.00 |
| 8% | 31.34 | 24.41 | 12.13 | 8.36 | 7.34 |
| 10% | 32.27 | 25.36 | 13.22 | 9.65 | 8.78 |
| 12% | 33.21 | 26.33 | 14.35 | 11.01 | 10.29 |

**TABLE 8.6** Monthly payments on a $1,000 loan.

**GENERAL RULE FOR COMPUTING $P(F|E)$** If $E$ and $F$ are events in a sample space, then $P(F|E) = \dfrac{P(E \cap F)}{P(E)}$.

**DEFINITION** In a weighted voting system, a voter's **Banzhaf power index*** is defined as

$$\frac{\text{the number of times the voter is critical in winning coalitions}}{\text{the total number of times voters are critical in winning coalitions}}.$$

**FINDING THE PRESENT VALUE OF AN ANNUITY** Assume that you are making $m$ periodic payments per year for $n$ total payments into an annuity that pays an annual interest rate of $r$. Also assume that each of your payments is $R$. Then to find the present value of your annuity, solve for $P$ in the equation

$$P\left(1+\frac{r}{m}\right)^n = R\left(\frac{\left(1+\frac{r}{m}\right)^n - 1}{\frac{r}{m}}\right).$$

**THE COMPOUND INTEREST FORMULA** Assume that an account with principal $P$ is paying an annual interest rate $r$ and compounding is being done $m$ times per year. If the money remains in the account for $n$ time periods, then the future value, $A$, of the account is given by the formula

$$A = P\left(1+\frac{r}{m}\right)^n.$$

Notice that in this formula, we have replaced $r$ by $\frac{r}{m}$, which is the annual rate divided by the number of compounding periods per year, and $t$ by $n$, which is the number of compounding periods.

**RULE FOR COMPUTING THE PROBABILITY OF A UNION OF TWO EVENTS** If $E$ and $F$ are events, then

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

If $E$ and $F$ have no outcomes in common, they are called *mutually exclusive events.* In this case, because $E \cap F = \varnothing$, the preceding formula simplifies to

$$P(E \cup F) = P(E) + P(F).$$

**DEFINITION** Assume that an experiment has outcomes numbered 1 to $n$ with probabilities $P_1, P_2, P_3, \ldots, P_n$. Assume that each outcome has a numerical value associated with it and these are labeled $V_1, V_2, V_3, \ldots, V_n$. The **expected value** of the experiment is

$$(P_1 \cdot V_1) + (P_2 \cdot V_2) + (P_3 \cdot V_3) + \cdots + (P_n \cdot V_n).$$

**THE HUNTINGTON–HILL APPORTIONMENT PRINCIPLE** If states X and Y have already been allotted $x$ and $y$ representatives, respectively, then state X should be given an additional representative in preference to state Y provided that

$$\frac{(\text{population of Y})^2}{y \cdot (y + 1)} < \frac{(\text{population of X})^2}{x \cdot (x + 1)}$$

Otherwise, state Y should be given the additional representative. We will often refer to a number of the form $\frac{(\text{population of X})^2}{x \cdot (x + 1)}$ as a **Huntington–Hill number.**