Your answers must be entered in your Examination Blue Book; answers written on this exam will not be graded. For full credit, you must show complete, correct and legible work. Read carefully before you start working. No books or notes are allowed. Calculators are allowed, but phones, PDAs, music players, Apple watches, and other electronic devices are not.

Solve problem 1 below, and solve any 13 of problems 2–16 on the following pages. The problems are weighted equally during grading. If you solve more than 13 of problems 2–16, you must indicate clearly which 13 you want graded. Otherwise, the first 13 will be selected by your grader.

**Part I**

**Problem 1:** In the graph below, determine the number of even and odd vertices, respectively. Does the graph have a bridge?
Part II

Problem 2: Sam deposited $2,000 in an account with an annual interest rate of 2.4%, compounded quarterly. What is the value of the account after 8 years? Round your answer to nearest cent.

Problem 3: A snail climbs four feet up a tree during the day and slides three feet down at night. The tree is ten feet tall. How many days will it take the snail to reach the top of the tree for the first time?

Problem 4: Consider the following weighted voting system:

\[ [12 : 4, 7, 8, 3] \]

(a) Are there any winning coalitions? If so, list them.
(b) Is there a dictator? Explain your answer.

Problem 5: There are 300 girls and 120 boys in the student athletics program at Monterey High School. The program supervisor has already chosen 3 girls and 2 boys to serve on a six-person committee to evaluate proposed athletics guidelines.

(a) Find the Huntington-Hill number for each group.
(b) Use the Huntington-Hill apportionment principle to decide if the sixth member on the committee should be a girl or a boy.

Problem 6: Use a truth table to determine if the statements 
\[ (p \lor \sim q) \land \sim (p \land q) \] and 
\[ p \land (p \land q) \] are logically equivalent.

Problem 7: 84 people are voting on which restaurant is to cater their event. There are three options: A, B and C. The ballots were submitted as follows:

<table>
<thead>
<tr>
<th></th>
<th>21</th>
<th>19</th>
<th>29</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>2nd</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>3rd</td>
<td>C</td>
<td>C</td>
<td>A</td>
<td>A</td>
</tr>
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(a) Which option wins when using the Borda count method?
(b) Which option wins when using plurality?
Problem 8: In (a)–(c), write the statement in symbolic form:

(a) The Democrats will control the Senate and environmental programs will not be cut. (Use $p$ for “The Democrats will control the Senate” and $q$ for “Environmental programs will be cut”.)

(b) If you exercise, then you will feel great or you will sleep better. (Use $p$ for “You exercise”, $q$ for “You will feel great” and $r$ for “You will sleep better”.)

(c) Press the Enter key if the screen is black and the Tab key if it is not. (Use $p$ for “The screen is black”, $q$ for “Press the Enter key” and $r$ for “Press the Tab key”.)

Problem 9: A traveling salesman lives in Buffalo (B) and wants to travel to Atlanta (A), Chicago (C), and Denver (D) to sell his saxophones. Using the Nearest Neighbor method on the graph below, what is the cheapest flight path to visit all three cities and return home? How much will this trip cost?

Problem 10: A pair of ordinary dice is rolled, and the numbers showing on the top sides are added together.

(a) What is the probability of getting a sum strictly less than 4?

(b) Using your understanding of the complement of an event, find the probability of getting a sum greater than or equal to 4.

Keep the answers as fractions.

Problem 11: Leo wants to save up for retirement, so he sets up an ordinary annuity. If he makes monthly payments of $600 and the annuity has an annual interest rate of 6%, compounded monthly, how much money will be in his account after 25 years? Round your answer to nearest cent.
Problem 12: Find the mean, median, mode, range and standard deviation of the following distribution:
45, 12, 50, 18, 30, 28, 12, 40, 33, 21, 37
In your answers and calculations, you may round numbers to two decimal places.

Problem 13: Below are the probabilities and values associated with five outcomes of an experiment. Calculate the expected value for the experiment.

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<th>Outcome</th>
<th>Probability</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.2</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>0.2</td>
<td>8</td>
</tr>
<tr>
<td>C</td>
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<td>-1</td>
</tr>
<tr>
<td>D</td>
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</tr>
<tr>
<td>E</td>
<td>0.2</td>
<td>-3</td>
</tr>
</tbody>
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Problem 14: A city consists of four districts with the following populations:
- North: 4,180
- South: 5,320
- East: 1,500
- West: 7,050
Use Hamilton’s apportionment method to assign 20 city council seats to the four districts.

Problem 15: Consider a normal distribution with a mean of 30 and a standard deviation of 6, and answer the following questions:
(a) What z-score corresponds to the raw score of 60?
(b) What is the raw score corresponding to a z-score of 1.5?

Problem 16: Bruce finally decided to buy a new house. While browsing online, he found a unique beach condo for sale in Galveston for only $229,000. The International Bank of Statistics offers him a 30-year home loan at 7.5% interest rate, compounded monthly since he will be making monthly payments. Bruce impulsively purchases the home. How much will his monthly payments be? Round your answer to nearest cent.
<table>
<thead>
<tr>
<th>Method</th>
<th>How the Winning Candidate Is Determined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plurality</td>
<td>The candidate receiving the most votes wins.</td>
</tr>
<tr>
<td>Borda count</td>
<td>Voters rank all candidates by assigning a set number of points to first choice, second choice, third choice, and so on; the candidate with the most points wins.</td>
</tr>
<tr>
<td>Plurality-with-elimination</td>
<td>Successive rounds of elections are held, with the candidate receiving the fewest votes being dropped from the ballot each time, until one candidate receives a majority of votes.</td>
</tr>
<tr>
<td>Pairwise comparison</td>
<td>Candidates are compared in pairs, with a point being assigned the voters’ preference in each pair. (In the case of a tie, each candidate gets a half point.) After all pairs of candidates have been compared, the candidate receiving the most points wins.</td>
</tr>
</tbody>
</table>

**THE HUNTINGTON–HILL APPORTIONMENT PRINCIPLE** If states X and Y have already been allotted $x$ and $y$ representatives, respectively, then state X should be given an additional representative in preference to state Y provided that

$$
\frac{(\text{population of Y})^2}{y \cdot (y + 1)} < \frac{(\text{population of X})^2}{x \cdot (x + 1)}
$$

Otherwise, state Y should be given the additional representative. We will often refer to a number of the form $\frac{(\text{population of X})^2}{x \cdot (x + 1)}$ as a Huntington–Hill number.

**FORMULA FOR FINDING THE FUTURE VALUE OF AN ORDINARY ANNUITY** Assume that we are making $n$ regular payments, $R$, into an ordinary annuity. The interest is being compounded $m$ times a year and deposits are made at the end of each compounding period. The future value (or amount), $A$, of this annuity at the end of the $n$ periods is given by the equation

$$
A = R \frac{\left(1 + \frac{r}{m}\right)^n - 1}{\frac{r}{m}}.
$$
THE COMPOUND INTEREST FORMULA Assume that an account with principal \( P \) is paying an annual interest rate \( r \) and compounding is being done \( m \) times per year. If the money remains in the account for \( n \) time periods, then the future value, \( A \), of the account is given by the formula

\[
A = P \left( 1 + \frac{r}{m} \right)^n.
\]

Notice that in this formula, we have replaced \( r \) by \( \frac{r}{m} \), which is the annual rate divided by the number of compounding periods per year, and \( t \) by \( n \), which is the number of compounding periods.

FORMULA FOR FINDING PAYMENTS ON AN AMORTIZED LOAN Assume that you borrow an amount \( P \), which you will repay by taking out an amortized loan. You will make \( m \) periodic payments per year for \( n \) total payments and the annual interest rate is \( r \). Then, you can find your payment by solving for \( R \) in the equation

\[
P \left( 1 + \frac{r}{m} \right)^n = R \left( \frac{\left( 1 + \frac{r}{m} \right)^n - 1}{\frac{r}{m}} \right)^*.
\]

FORMULA FOR CONVERTING RAW SCORES TO \( z \)-SCORES Assume a normal distribution has a mean of \( \mu \) and a standard deviation of \( \sigma \). We use the equation

\[
z = \frac{x - \mu}{\sigma}
\]

to convert a value \( x \) in the nonstandard distribution to a \( z \)-score.

DEFINITION Assume that an experiment has outcomes numbered 1 to \( n \) with probabilities \( P_1, P_2, P_3, \ldots, P_n \). Assume that each outcome has a numerical value associated with it and these are labeled \( V_1, V_2, V_3, \ldots, V_n \). The expected value of the experiment is

\[
(P_1 \cdot V_1) + (P_2 \cdot V_2) + (P_3 \cdot V_3) + \cdots + (P_n \cdot V_n).
\]
Problem 1: In the graph below, determine the number of even and odd vertices, respectively. Does the graph have a bridge?

![Graph with vertices labeled 1 to 6]

Answer: There are 2 even and 4 odd vertices. The edge connecting vertices 4 and 6 is a bridge.

Problem 2: Sam deposited $2,000 in an account with an annual interest rate of 2.4%, compounded quarterly. What is the value of the account after 8 years? Round your answer to nearest cent.

Answer: We have

\[ P = 2000, \quad r = 0.024, \quad m = 4, \quad \text{and} \quad t = 8, \]

which gives us

\[ A = 2000 \times \left(1 + \frac{0.024}{4}\right)^{4 \cdot 8} = 2000 \times (1.006)^{32} = 2421.95 \]

Problem 3: A snail climbs four feet up a tree during the day and slides three feet down at night. The tree is ten feet tall. How many days will it take the snail to reach the top of the tree for the first time?
Answer: The best way to find the solution to this problem is to do it step by step. During day 1, the snail climbs 4 feet and then slides 3 feet down. Therefore, the second day the snail starts 1 foot higher. During the second day the snail will reach the 5’ mark of the tree and then slide down again. On the third day the snail will reach 6’ mark. And so on. The snail will begin the 7th day of its journey at the 6’ mark, and by climbing 4 feet that day, the snail will reach the top of the tree. The answer is therefore 7 days.

Problem 4: Consider the following weighted voting system:

\[ [12 : 4, 7, 8, 3] \]

(a) Are there any winning coalitions? If so, list them.
(b) Is there a dictator? Explain your answer.

Answer: (a) Yes, there are
\{4, 8\}, \{7, 8\}, \{4, 7, 8\}, \{4, 7, 3\}, \{4, 8, 3\}, \{7, 8, 3\}, and \{4, 7, 8, 3\}

(b) No: None of the voters have weight 12 or above.

Problem 5: There are 300 girls and 120 boys in the student athletics program at Monterey High School. The program supervisor has already chosen 3 girls and 2 boys to serve on a six-person committee to evaluate proposed athletics guidelines.

(a) Find the Huntington-Hill number for each group.
(b) Use the Huntington-Hill apportionment principle to decide if the sixth member on the committee should be a girl or a boy.

Answer: (a) For the girls, the Huntington-Hill number is
\[
\frac{300^2}{3(3 + 1)} = \frac{300^2}{12} = 7500
\]
and for the boys it is
\[
\frac{120^2}{2(2 + 1)} = \frac{120^2}{6} = 2400
\]

(b) Since 7500 > 2400, the sixth member should be a girl.
Problem 6: Use a truth table to determine if the statements
\((p \lor \sim q) \land \sim (p \land q)\) and \(p \land (p \land q)\) are logically equivalent.

Answer: We get the tables

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<td>p</td>
<td>q</td>
<td>p ∨ ∼ q</td>
<td>p ∧ q</td>
<td>(p ∨ ∼ q) ∧ ∼ (p ∧ q)</td>
<td>p ∧ (p ∧ q)</td>
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Clearly, the two statements are not logically equivalent.

Problem 7: 84 people are voting on which restaurant is to cater their event. There are three options: A, B and C. The ballots were submitted as follows:

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<td>C</td>
<td>C</td>
<td>A</td>
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</table>

(a) Which option wins when using the Borda count method?
(b) Which option wins when using plurality?

Answer: (a) A gets \(21 \cdot 3 + 19 \cdot 2 + 29 + 15 = 145\) points. B gets \(19 \cdot 3 + 15 \cdot 3 + 21 \cdot 2 + 29 \cdot 2 = 202\) points. C gets \(29 \cdot 3 + 15 \cdot 2 + 21 + 19 = 157\) points. Therefore, B wins.

(c) B gets the largest number of first choice votes. Therefore, B wins (again).

Problem 8: In (a)–(c), write the statement in symbolic form:

(a) The Democrats will control the Senate and environmental programs will not be cut. (Use \(p\) for “The Democrats will control the Senate” and \(q\) for “Environmental programs will be cut”.)

(b) If you exercise, then you will feel great or you will sleep better. (Use \(p\) for “You exercise”, \(q\) for “You will feel great” and \(r\) for “You will sleep better”.)

(c) Press the Enter key if the screen is black and the Tab key if it is not. (Use \(p\) for “The screen is black”, \(q\) for “Press the Enter key” and \(r\) for “Press the Tab key”.)
Answer: (a) $p \land \sim q$

(b) $p \Rightarrow (q \lor r)$

(c) $(p \Rightarrow q) \land (\sim p \Rightarrow r)$

Problem 9: A traveling salesman lives in Buffalo (B) and wants to travel to Atlanta (A), Chicago (C), and Denver (D) to sell his saxophones. Using the Nearest Neighbor method on the graph below, what is the cheapest flight path to visit all three cities and return home? How much will this trip cost?

![Graph](image)

Answer: The path produced by Nearest Neighbor is BDACB. It costs $980.

Problem 10: A pair of ordinary dice is rolled, and the numbers showing on the top sides are added together.

(a) What is the probability of getting a sum strictly less than 4?

(b) Using your understanding of the complement of an event, find the probability of getting a sum greater than or equal to 4.

Keep the answers as fractions.
Answer: (a) The set of outcomes with sum strictly less than 4 is 
\[ A = \{(1, 1), (1, 2), (2, 1)\} \]
The set \( S \) of all outcomes has 36 elements. Therefore, the probability is 
\[ P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12} \]

(b) The set of outcomes with sum greater than or equal to 4 is the complement \( A' \) of \( A \). Therefore,
\[ P(A') = 1 - P(A) = \frac{11}{12} \]

Problem 11: Leo wants to save up for retirement, so he sets up an ordinary annuity. If he makes monthly payments of $600 and the annuity has an annual interest rate of 6%, compounded monthly, how much money will be in his account after 25 years? Round your answer to nearest cent.

Answer: We have 
\[ R = 600, \quad r = 0.06, \quad m = 12, \quad \text{and} \quad t = 25 \]
This gives us
\[ A = 600 \times \frac{(1 + 0.06/12)^{12 \times 25} - 1}{0.06/12} = 600 \times \frac{1.005^{300} - 1}{0.005} = 415,796.38 \]

Problem 12: Find the mean, median, mode, range and standard deviation of the following distribution:
45, 12, 50, 18, 30, 28, 12, 40, 33, 21, 37
In your answers and calculations, you may round numbers to two decimal places.
Answer: We start by sorting the numbers:

\[
12, 12, 18, 21, 28, 30, 33, 37, 40, 45, 50
\]

We then see that the range is \(50 - 12 = 38\), the mean is

\[
\frac{12 + 12 + 18 + 21 + 28 + 30 + 33 + 37 + 40 + 45 + 50}{11} = \frac{326}{11} = 29.63636364 \approx 29.64,
\]

the median is 30, and the mode is 12.

For the standard deviation, we first find

\[
\begin{array}{c|c|c}
 x & x - \bar{x} & (x - \bar{x})^2 \\
\hline
12 & 12 - 29.64 = -17.64 & 311.17 \\
12 & 12 - 29.64 = -17.64 & 311.17 \\
18 & 18 - 29.64 = -11.64 & 135.49 \\
21 & 21 - 29.64 = -8.64 & 74.65 \\
28 & 28 - 29.64 = -1.64 & 2.69 \\
30 & 30 - 29.64 = 0.36 & 0.13 \\
33 & 33 - 29.64 = 3.36 & 11.29 \\
37 & 37 - 29.64 = 7.36 & 54.17 \\
40 & 40 - 29.64 = 10.36 & 107.33 \\
45 & 45 - 29.64 = 15.36 & 235.93 \\
50 & 50 - 29.64 = 20.36 & 414.53 \\
\end{array}
\]

and

\[
\Sigma(x - \bar{x})^2 = 1658.55,
\]

giving us

\[
\sigma^2 = \frac{\Sigma(x - \bar{x})^2}{n - 1} \quad \text{and} \quad \sigma = \sqrt{\frac{1658.55}{11 - 1}} = \sqrt{\frac{1658.55}{10}} = \sqrt{165.85} \approx 12.88
\]

Problem 13: Below are the probabilities and values associated with five outcomes of an experiment. Calculate the expected value for the experiment.

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<thead>
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<th>Value</th>
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<td>2</td>
</tr>
<tr>
<td>E</td>
<td>0.2</td>
<td>-3</td>
</tr>
</tbody>
</table>
Answer: We get an expected value of
\[ 0.2 \times 4 + 0.2 \times 8 + 0.1 \times (-1) + 0.3 \times 2 + 0.2 \times (-3) = 2.3 \]

**Problem 14:** A city consists of four districts with the following populations:
- North: 4,180
- South: 5,320
- East: 1,500
- West: 7,050

Use Hamilton’s apportionment method to assign 20 city council seats to the four districts.

**Answer:** The standard divisor is
\[ \frac{4180 + 5320 + 1500 + 7050}{20} = 902.5 \]

We then get the table

<table>
<thead>
<tr>
<th></th>
<th>Standard quota</th>
<th>Lower quota</th>
<th>Fractional part</th>
<th>Apportionment</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>4180/902.5 = 4.63</td>
<td>4</td>
<td>0.63</td>
<td>4</td>
</tr>
<tr>
<td>South</td>
<td>5230/902.5 = 5.89</td>
<td>5</td>
<td>0.89</td>
<td>6</td>
</tr>
<tr>
<td>East</td>
<td>1500/902.5 = 1.66</td>
<td>1</td>
<td>0.66</td>
<td>2</td>
</tr>
<tr>
<td>West</td>
<td>7050/902.5 = 7.81</td>
<td>7</td>
<td>0.81</td>
<td>8</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>17</strong></td>
<td></td>
<td><strong>20</strong></td>
</tr>
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</table>

**Problem 15:** Consider a normal distribution with a mean of 30 and a standard deviation of 6, and answer the following questions:

(a) What z-score corresponds to the raw score of 60?
(b) What is the raw score corresponding to a z-score of 1.5?
Answer: Note that $\mu = 30$ and $\sigma = 6$.

(a) We have

$$Z = \frac{x - \mu}{\sigma} = \frac{60 - 30}{6} = \frac{30}{6} = 5$$

(b) We have

$$Z = 1.5 = \frac{x - \mu}{\sigma} = \frac{x - 30}{6},$$

giving us

$$x = 39$$

Problem 16: Bruce finally decided to buy a new house. While browsing online, he found a unique beach condo for sale in Galveston for only $229,000. The International Bank of Statistics offers him a 30-year home loan at 7.5% interest rate, compounded monthly since he will be making monthly payments. Bruce impulsively purchases the home. How much will his monthly payments be? Round your answer to nearest cent.

Answer: We use the formula for an amortized loan. We have the principal $P$ as $229,000$, the interest rate $r$ as $7.5\% = 0.075$, $m = 12$ and $n = 12 \cdot 30 = 360$. The formula

$$P(1 + \frac{r}{m})^n = R\left(\frac{(1 + \frac{r}{m})^n - 1}{\frac{r}{m}}\right)$$

then becomes

$$\$229,000 \times \left(1 + \frac{0.075}{12}\right)^{360} = R\left(\frac{(1 + \frac{0.075}{12})^{360} - 1}{\frac{0.075}{12}}\right)$$

which we solve to find

$$R = \$1601.20$$