Final Exam – Math 1321 – Spring 2011

1. Perform the indicated operations:
   a) Find the complement and supplement of $71°$.
   b) Compute $90° - 72°\,15'$
   c) Convert to decimal degrees $32°\,45'$
   d) Convert to radians $225°$
   e) Find the reference angle for $-171°$.
   f) Convert $5\pi/3$ radians to degrees

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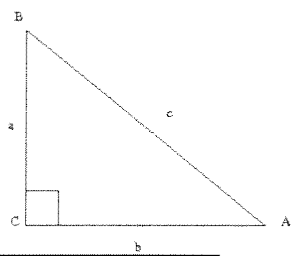
2. Solve the right triangle given $c = 650$, $A = 25°$, i.e., find a, b and B. Round answers to 2 digits after the decimal point.



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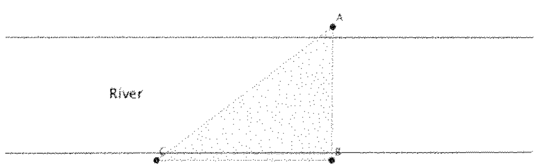
3. A surveyor needs to find the distance across a straight river. Markers are set up at points A and B on opposite sides of the river (see diagram), and at point C that is 260 feet from B, the angle at C is measured and found to be $37°$. The angle at B is a right angle. Find the distance from A to B.



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4. A student walking towards a radio tower observes that the top has an angle of elevation of $45°$. When she walks 100 feet closer, the angle is $60°$. How tall is the tower?

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5. Given a circle of radius 5 cm. Find
   a) The radian measure of an arc of length 8 cm.
   b) The area of a sector having angle $30°$.
   c) The linear speed of a point moving on the circle with angular speed $\omega = \pi/6$ radian/sec.

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6. Find the exact trig function values
   a) $\tan(5\pi/6)$ b) $\cos(-7\pi/3)$ c) $\sec(13\pi/3)$

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7. Find the length of the arc traversed by the tip of an 8 inch clock hand as it moves 10 minutes.

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8. (a) Find the equation of a sine wave that is obtained by shifting the graph of $y = \sin(x)$ to the right 6 units and downward 2 units and is vertically stretched by a factor of 3 when compared to $y = \sin(x)$.
   (b) Sketch the graph of $y = 1 - 2\cos(2x)$ over one full period.

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9. Verify the identity for all t for which the expressions exists
$$\frac{\cos(t)}{1-\sin(t)} - \tan(t) = \frac{1}{\cos(t)}.$$

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10. Given $0 < \alpha < \pi/2$ with $\cos(\alpha) = \frac{3}{5}$ and β in the 3rd quadrant with $\sin(\beta) = -\frac{5}{7}$. Find the exact values of (a) $\sin(\alpha - \beta)$, (b) $\cos(\alpha + \beta)$.

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11. Find the exact value of
    (a) $\cos\left(\frac{7\pi}{8}\right)$,
    (b) $\sin(15°)$.

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12. Solve the equation for all θ in $[0, 2\pi]$.
    (a) $2\cos^2(\theta) - \cos(\theta) - 1 = 0$.
    (b) $\sin(2\theta) - \cos(\theta) = 0$.

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13. Evaluate :
    (a) $\cos^{-1}(\sin(5\pi/4))$, (b) $\sin(\cos^{-1}(1/3))$.

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14. Solve the triangle $\triangle ABC$: (a) If $A = 60°$, $b = 6$ and $c = 9$ find a, B and C. (b) If $a = 7$, $C = 40°$, $A = 30°$, find B, b and c.

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15. Solve the following:
    (a) The vector $\boldsymbol{v}$ has magnitude $|\boldsymbol{v}| = 50$ and direction $\alpha = 30°$. Find the horizontal and vertical components of $\boldsymbol{v}$.
    (b) Find the angle between the vectors $\langle 1, 2\rangle$ and $\langle -3, 2\rangle$.

Solutions for Final Exam – Math 1321 – Spring 2011

1. Perform the indicated operations:
   a) Find the complement and supplement of $71°$.
   b) Compute $90° - 72°\,45'$
   c) Convert to decimal degrees $32°\,45'$
   d) Convert to radians $225°$
   e) Find the reference angle for $-171°$.
   f) Convert $5\pi/3$ radians to degrees

   Answers:

   (a) Complement $19°$; Supplement 109
   (b) $17°\,15'$
   (c) $32.75°$
   (d) $5\pi/4$
   (e) $9°$
   (f) $300°$

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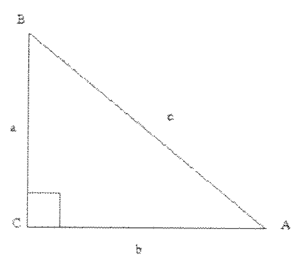
2. Solve the right triangle given $c = 650$, $A = 25°$, i.e., find a, b and B. Round answers to 2 digits after the decimal point.



   Answers:

   (a) $B = 90° - 25° = 65°$
   (b) $a = 650\sin(25°) = 274.71$
   (c) $b = 650\cos(25°) = 589.10$

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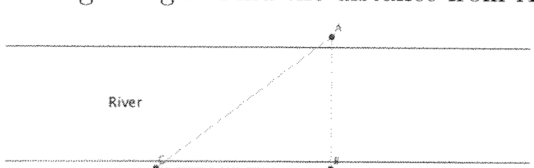
3. A surveyor needs to find the distance across a straight river. Markers are set up at points A and B on opposite sides of the river (see diagram), and at point C that is 260 feet from B, the angle at C is measured and found to be $37°$. The angle at B is a right angle. Find the distance from A to B.



   Answers:

   distance $= 260\,\tan(37°) = 195.92$ ft.

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4. A student walking towards a radio tower observes that the top has an angle of elevation of $45°$. When she walks 100 feet closer, the angle is $60°$. How tall is the tower?

   Answers:
   Let y denote the height of tower and $x + 100$ the intial distance from the tower. Then we have

   $1 = \tan(45°) = \dfrac{y}{100+x}$ and $\sqrt{3} = \tan(60°) = \dfrac{y}{x}$
   $\Rightarrow y = \dfrac{100\sqrt{3}}{\sqrt{3}-1} \approx 236.60$ feet.

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5. Given a circle of radius 5 cm. Using two digits of accuracy ind
   a) The radian measure of an arc of length 8 cm.
   b) The area of a sector having angle $30°$.
   c) The linear speed of a point moving on the circle with angular speed $\omega = \pi/6$ radian/sec.

   Answers:

   (a) $\theta = s/r = 8/5 = 1.6$ radians
   (b) $A = 1/2r^2\theta = \dfrac{(5^2)(30°)(\pi/180°)}{2} = 6.54\text{cm}^2$
   (c) linear speed $\nu = \dfrac{s}{t} = r\omega = 5\pi/6$ cm/sec

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6. Find the exact trig function values
   a) $\tan(5\pi/6)$ b) $\cos(-7\pi/3)$ c) $\sec(13\pi/3)$

   Answers:

   (a) $\tan(5\pi/6) = -\dfrac{\sin(\pi/6)}{\cos(\pi/6)} = -\dfrac{1}{\sqrt{3}}$
   (b) $\cos(-7\pi/3) = \cos(\pi/3) = \dfrac{1}{2}$
   (c) $\sec(13\pi/3) = 1/\cos(\pi/3) = 2$

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7. Find the length of the arc traversed by the tip of an 8 inch clock hand as it moves 10 minutes.

   Answers:
   10 minutes is 1/6th of the distance around the circle which gives $\theta = (1/6)2\pi = \pi/3$ radians. So we have $s = r\theta = 8\pi/3 = 8.38$ in .

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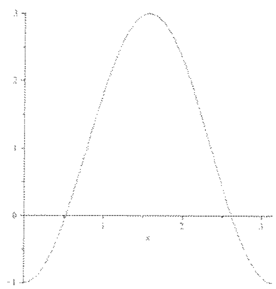
8. (a) Find the equation of a sine wave that is obtained by shifting the graph of $y = \sin(x)$ to the right 6 units and downward 2 units and is vertically stretched by a factor of 3 when compared to $y = \sin(x)$.
(b) Sketch the graph of $y = 1 - 2\cos(2x)$ over one full period.

(a) $y = 3\sin(x - 6) - 2$

(b) 

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9. Verify the identity for all t for which the expressions exists

$$\frac{\cos(t)}{1-\sin(t)} - \tan(t) = \frac{1}{\cos(t)}.$$

Answers:

$$\frac{\cos(t)}{1-\sin(t)} - \tan(t) = \frac{\cos^2(t) - \sin(t)(1-\sin(t))}{\cos(t)(1-\sin(t))}$$
$$= \frac{1-\sin(t)}{\cos(t)(1-\sin(t))} = \frac{1}{\cos(t)}$$

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10. Given $0 < \alpha < \pi/2$ with $\cos(\alpha) = \frac{3}{5}$ and β in the 3rd quadrant with $\sin(\beta) = -\frac{5}{7}$. Find the exact values of (a) $\sin(\alpha - \beta)$, (b) $\cos(\alpha + \beta)$.

Answers:
First we have $\sin(\alpha) = \sqrt{1-(3/5)^2} = 4/5$ and $\cos(\beta) = -\sqrt{1-(5/7)^2} = -2\sqrt{6}/7$

(a) $\sin(\alpha-\beta) = (4/5)(-2\sqrt{6}/7) - (-5/7)(3/5) = \dfrac{-8\sqrt{6}+15}{35}$

(b) $\cos(\alpha-\beta) = (3/5)(-2\sqrt{6}/7) + (4/5)(-5/7) = \dfrac{-6\sqrt{6}-20}{35}$

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11. Find the exact value of

(a) $\cos\left(\frac{7\pi}{8}\right)$,
(b) $\sin(15°)$.

Answers:

(a) $\cos\left(\frac{7\pi}{8}\right) = -\sqrt{\frac{1+\cos(7\pi/4)}{2}} = -\sqrt{\frac{1+\cos(\pi/4)}{2}} = -\frac{\sqrt{2+\sqrt{2}}}{2}$

(b) $\sin(15°) = \sin\left(\frac{\pi/6}{2}\right) = \frac{\sqrt{2-\sqrt{3}}}{2}$

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12. Solve the equation for all θ in $[0, 2\pi]$.

(a) $2\cos^2(\theta) - \cos(\theta) - 1 = 0$.
(b) $\sin(2\theta) - \cos(\theta) = 0$.

Answers:

(a) $(2\cos(\theta)+1)(\cos(\theta)-1) = 0$ which implies $\cos(\theta) = -1/2$ and $\cos(\theta) = 1$. So we have
$\theta = 2\pi/3, 4\pi/3, 0, 2\pi$

(b) $2\sin(\theta)\cos(\theta) - \cos(\theta) = 0$ which implies $\sin(\theta) = 1/2$ and $\cos(\theta) = 0$. So we have
$\theta = \pi/2, 3\pi/2, \pi/6, 5\pi/6$

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13. Evaluate :

(a) $\cos^{-1}(\sin(5\pi/4))$, (b) $\sin(\cos^{-1}(1/3))$.

Answers:

(a) $\sin(5\pi/4) = -\sqrt{2}/2$ and $\cos^{-1}\left(-\sqrt{2}/2\right) = 3\pi/4$

(b) $\sin(\cos^{-1}(1/3)) = 2\sqrt{2}/3$

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14. Solve the triangle $\triangle ABC$: (a) If $A = 60°$, $b = 6$ and $c = 9$ find a, B and C. (b) If $a = 7$, $C = 40°$, $A = 30°$, find B, b and c.

Answers:

(a) $a^2 = 6^2 + 9^2 - 2 \cdot 6 \cdot 9 \cdot \cos(60^\circ) = 63$ so $a = \sqrt{63} = 3\sqrt{7} \approx 7.94$.
$B = \sin^{-1}\left(b\,\frac{\sin(A)}{a}\right) = \sin^{-1}\left(6\,\frac{\sin(60^\circ)}{3\sqrt{7}}\right) = \sin^{-1}\left(\sqrt{3/7}\right) \approx 40.89^\circ$. And finally $C = 180^\circ - A - C = 79.11^\circ$.

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15. Solve the following:
(a) The vector $\boldsymbol{v}$ has magnitude $|\boldsymbol{v}| = 50$ and direction $\alpha = 30^\circ$. Find the horizontal and vertical components of $\boldsymbol{v}$.
(b) Find the angle between the vectors $\langle 1, 2\rangle$ and $\langle -3, 2\rangle$. Given answer in decimal degrees.

Answers:

(a) Let $\boldsymbol{v} =< x, y >$ then
$x = 50\cos(30^\circ) = 25\sqrt{3}$ and
$y = 50\sin(30^\circ) = 25$.

(b) $\cos(\theta) = \dfrac{\langle 1, 2\rangle \cdot \langle -3, 2\rangle}{|\langle 1, 2\rangle|\,|\langle -3, 2\rangle|} = \dfrac{1}{\sqrt{5}\sqrt{13}}$

So we have $\theta = \cos^{-1}\left(\frac{1}{\sqrt{65}}\right)\left(\frac{180^\circ}{\pi}\right) \approx 82.87^\circ$.
