Math 1321 **Trigonometry Final Exam** **Fall 2014**

Instructions: ***Show all work and answers in your blue book. Failure to show all work may result no credit for that problem. Some formulas are given on the back of this sheet.***

1. Let t be an angle between $\pi/2$ and π such that $\sin t = 5/13$. Find:
   a) $\csc(t)$
   b) $\sin(2t)$
   c) $\sin(-t) + \cos(-t)$

2. a) Write a $\cos(75°)$ in terms of the cofunction of a complementary angle.
   b) Without a calculator, find the exact value of $\cos(75°)$.

3. An airplane is flying at an altitude of 10200 feet above level ground. The angle of depression from the plane to an airport is 12°. To the nearest mile, find the distance from the plane to the airport. (Use 1 mile = 5280 feet)

4. Consider the function $y = 4 - 2\cos(3x + \pi)$.
   a) What is its period?
   b) What is the amplitude of its graph?
   c) What is the vertical shift of its graph?
   d) What is the y-intercept of its graph?
   e) What is its phase shift?

5. A carousel of radius 20 feet is rotating at 2.5 revolutions per minute. Find the linear speed of a point on the perimeter of this carousel in units of feet per minute.

6. Suppose a cable, wrapped around a circular cylinder of radius 2 meters, unwinds as the cylinder rotates. Through what angle (in degrees) does the cylinder rotate if 5π meters of cable are released?

7. Find all values x in $[0°, 180°]$ such that $\tan(2x) = 1$.

8. Find all angles θ in $[0, 2\pi)$ for which $\sin\theta = 1 - 2\sin^2\theta$.

9. a) Why is $\arcsin(-2)$ not defined?
   b) Give the exact radian measure of $\arccos(-\sqrt{2}/2)$.
   c) Simplify the expression $\tan(\cos^{-1}\left(\frac{x}{5}\right))$. Your answer should be an algebraic expression in x.

10. Verify the identity:

$$\frac{\sin(2x)}{1 - \cos(2x)} = \cot(x)$$

11. Two bicyclists leave from the same location at the same time, traveling on courses that have an angle of 75° between them. The first cyclist travels at constant rate of 20 miles per hour and the other travels at 25 miles per hour. To the nearest mile, how far apart will the bicyclists be after two hours?

12. Use an identity to find the exact value of

$$\frac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{12}\right)}{1 + \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{12}\right)}$$

13 Let $\boldsymbol{u} = \langle 6, -8 \rangle$, $\boldsymbol{w} = \langle 2, -5 \rangle$, and $\boldsymbol{v} = \langle -3, 7 \rangle$.

a) Find $|\boldsymbol{u}|$
b) Find $\boldsymbol{w} - \boldsymbol{v}$
c) Find $\boldsymbol{u} \cdot (\boldsymbol{v} + 2\boldsymbol{w})$
d) Find the measure of the angle between **u** and **w** to the nearest tenth of a degree.

14. Consider an oblique triangle ABC. Given that the measure of angle A is 45°, the length of side AC is 3 cm, and the length of side BC is $3\sqrt{2}$ cm, find the measure of angle B and the length of side AB. Rounding to one decimal place is sufficient.

Trig Identities and Formulas

Sum and Difference Formulas

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Double Angle Formulas

$$\cos(2A) = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\sin(2A) = 2\sin A \cos A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Half-Angle Formulas

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$$

Law of Sines

In triangle ABC with sides a, b, c:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

In triangle ABC with sides a, b, c:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$