MATH 1321 Final Exam Spring 2014 Version 1

Instructions: **Show all of your work in the blue book. Failure to show all work may result in no credit for that problem.**

1. A windmill with radius 14 feet makes 130 revolutions per minute. What is the linear speed of a point on the edge of the windmill in feet per second?

2. If $y = 5 + 3\cos(3x - 6)$, find the:
   a) amplitude; b) vertical shift;
   c) period; d) phase shift

3. Find the exact value of each of the following without using a calculator.
   a) $\arccos(\frac{-1}{2})$; b) $\cos(\arcsin\frac{3}{5})$;
   c) $\sin(\frac{5\pi}{12})$; d) $\frac{2\tan(\frac{\pi}{12})}{1-\tan^2(\frac{\pi}{12})}$

4. For the vectors $\mathbf{u} = \langle 7, -1\rangle$ and $\mathbf{v} = \langle 1, 1\rangle$, find
   a) $2\mathbf{u} - 3\mathbf{v}$
   b) $\mathbf{u} \cdot \mathbf{v}$
   c) $|3\mathbf{u}|$

5. The base of a tree is 32.5 meters away from a building. Find the height of the building if the angle of elevation from the base of the tree to the top of the building is $40°18'$.

6. Verify that the following equation is an identity: $(\sin(x) + \cos(x))^2 = \sin(2x) + 1$.

7. Find the area of the sector of a circle with radius 5.4 feet and a central angle of $120°$.

8. Find all solutions to the equation $\sin^2(\theta) - 3\sin(\theta) + 2 = 0$ in the domain $[0, 2\pi)$.

9. Find the remaining angles and sides of triangle ABC provided that $C = 84°$, $A = 73°$, and $a = 128$ inches.

10. A spool of thread has a diameter of 12 cm. If a piece of string is wound around through an angle of $540°$, how much thread is on the spool?

11. A fishing boat leaves port headed due north and travels 14.1 mi. The boat then turns on a bearing of N36°E and travels 3.8 mi. How far is the fishing boat from port?

12. A stone glass window is formed by piecing together 20 equilateral triangles, each with side length of 4 inches. What is the area of the stone glass window?

Trigonometric Identities and Forumulas

Sum and Difference Formulas

$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$

$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$

$\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$

Double Angle Formulas

$\cos(2A) = \cos^2(A) - \sin^2(A) = 2\cos^2(A) - 1 = 1 - 2\sin^2(A)$

$\sin(2A) = 2\sin(A)\cos(A)$

$\tan(2A) = \frac{2\tan(A)}{1-\tan^2(A)}$

Half-Angle Identities

$\cos(\frac{A}{2}) = \pm\sqrt{\frac{1+\cos(A)}{2}}$

$\sin(\frac{A}{2}) = \pm\sqrt{\frac{1-\cos(A)}{2}}$

$\tan(\frac{A}{2}) = \pm\sqrt{\frac{1-\cos(A)}{1+\cos(A)}} = \frac{\sin(A)}{1+\cos(A)} = \frac{1-\cos(A)}{\sin(A)}$

Law of Sines

For triangle ABC, $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

Law of Cosines

For triangle ABC, the following hold:

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$b^2 = a^2 + c^2 - 2ac\cos(B)$

$c^2 = a^2 + b^2 - 2ab\cos(C)$

Area of a Triangle (SAS)

For triangle ABC, the area $\mathcal{A}$ is given by the following formulas:

$\mathcal{A} = \frac{1}{2}bc\sin(A)$

$\mathcal{A} = \frac{1}{2}ab\sin(C)$

$\mathcal{A} = \frac{1}{2}ac\sin(B)$

Heron's Area Formula (SSS)

If a triangle has sides of lenghts a, b, and c, with semiperimeter $s = \frac{1}{2}(a+b+c)$ then the area $\mathcal{A}$ of the triangle is given by the following formula.

$$\mathcal{A} = \sqrt{s(s-a)(s-b)(s-c)}$$