Math 1321 Trigonometry Final Exam Spring 2015

<u>Instructions</u>: Show all work and answers in your blue book. Failure to show all work may result in no credit for that problem. If the question requires an exact value, then calculator values will not be accepted. Some formulas are given on the back of this sheet.

1. (*4 pts*) Convert $15\pi/4$ to degrees and give the quadrant of this angle.

2. (*4 pts*) Find the *exact value* of $4\sin\left(\frac{15\pi}{4}\right) + 2\cos\left(-\frac{7\pi}{6}\right)$.

3. (*6 pts*) The terminal side of an angle A in standard position is along the line $4x + y = 0$ with $x \le 0$. Give the value of $\sin A$ and of $\tan A$.

4. (*6 pts*) Suppose $\sin\theta > 0$ and $\cos\theta = -4/5$. Find the *exact value,* without using a calculator, of each of the following.
   (a) $\csc\theta$ (b) $\cos(-\theta)$ (c) $\cos(\pi/2 - \theta)$

5. (*6 pts*) A circular sector has an area of 50 square inches. The radius of the circle is 5 inches. What is the arc length of this sector?

6. (*6 pts*) Verify the identity $\tan x + \cot x = \sec x \csc x$.

7. (*6 pts*) A ladder leans against a building. The foot of the ladder is 6 feet from the building and the ladder makes a 60° with the level ground. How long is the ladder and to what height does the ladder reach on the building?

8. (*6 pts*) A blade on a wind turbine is 10 feet long and makes 25 revolutions per minute. Find the linear speed in feet per minute of the tip of the blade.

9. (*6 pts*) Consider the function $y = 5\sin(2x - \pi)$.
   a) What is the amplitude of its graph?
   b) What is its period?
   c) What is its phase shift?
   d) **BONUS**: For $\frac{\pi}{4} \le x \le \frac{3\pi}{4}$, solve the equation $y = 5\sin(2x - \pi)$ for x in terms of y.

10. (*8 pts*) a) Give all values of θ in $[0,2\pi)$ for which $\sec\theta = 2$. Give *exact radian* values only.
    b) Give all values of x in $[0,\ 360°)$ for which $\sin(2x) = 1$. Give *exact degree* values only.

11. (*6 pts*) Find all x in $[0, 2\pi)$ for which $\cos^2 x - \cos x = 2$.

12. (*9 pts*) Use identities to find the *exact value* of each of the following:
    a) $2\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$ b) $\tan(105°)$ *c*) $\cos(22.5°)$

13. (*6 pts*) Answer parts (a) – (c):
    a) Give the *exact* radian measure of $\arcsin(-\sqrt{3}/2)$.
    b) Give the *exact* value of $\sin\left(\cos^{-1}\left(\frac{1}{3}\right)\right)$

14. (*6 pts*) Triangle *ABC* has $\angle A = 32°$, $\angle B = 81.8°$, and side $a = 42.9$ inches. To the nearest tenth of an inch, find the length of side *c*.

15. (*6 pts*) Suppose that in triangle *ABC* none of the angles are right angles and the three sides have lengths of 4, 8, and 10 cm. To the nearest tenth of a degree, find the measure of the largest angle.

16. (*9 pts*) Consider the vectors $\boldsymbol{u} = \langle 6, -8 \rangle$ and $\boldsymbol{v} = \langle 2, -5 \rangle$.
   a) Find $|\,\boldsymbol{u}\,|$
   b) Find $\boldsymbol{u} \cdot (\boldsymbol{v} - 2\boldsymbol{u})$
   c) Find the measure of the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ to the nearest tenth of a degree.

Trig Identities and Formulas

Sum and Difference Formulas

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Half-Angle Formulas

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$$

Double Angle Formulas

$$\cos(2A) = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\sin(2A) = 2\sin A \cos A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Law of Sines

In triangle *ABC* with sides *a*, *b*, *c*:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

In triangle *ABC* with sides *a*, *b*, *c*:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$