

MATH 1321 - FINAL EXAMINATION
FALL 2016

SHOW ALL YOUR WORK. EACH PROBLEM IS WORTH THE SAME NUMBER OF POINTS.

1.
 - a. Find all acute angles θ such that $\theta + 15^\circ$ and $4\theta + 45^\circ$ are acute, and $\sin(\theta + 15^\circ) = \cos(4\theta + 45^\circ)$.
 - b. Find the value of $\cos(-930^\circ)$.
2. The angle of elevation from a point on the ground 15.5 m from the base of a tree to the top of the tree is 60.4° . Find the height of the tree.
3. Madison(South Dakota) and Lubbock(Texas) are 1200 km apart and lie on the same north-south line. The latitude of Madison is 44° N. What is the latitude of Lubbock? (Assume that the radius of the Earth is 6400 km.)
4. Earth revolves on its axis once every 24 hr. Assuming that Earth's radius is 6400 km, find the following:
 - a. Angular speed of the Earth in radians per hour.
 - b. Linear speed at the North pole or South pole.
 - c. Linear speed when the latitude is 45° .

Give your answers for the linear speed in kilometers per hour.

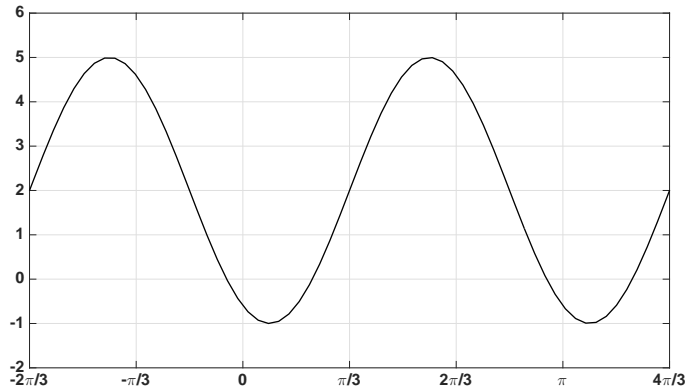
5. Suppose that the average monthly low temperatures for a small town are given in the table.

Month (x)	1	2	3	4	5	6	7	8	9	10	11	12
Temperature (y)	19	27	38	45	57	62	65	58	51	41	33	25

A model of these data is given by:

$$y = f(x) = 23 \sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 42$$

- a. What is the vertical displacement of y?
 - b. What is the amplitude of y?
 - c. What is the period of y?
 - d. What is the horizontal displacement (phase shift) of y?
6. The function graphed is of the form of $y = a \sin [b(x - d)] + c$, where a , b , c , and d are constants.



Write down the equation of the function graphed.

7. Suppose that A and B are angles in standard position with $\sin A = \frac{4}{5}$, $\frac{\pi}{2} < A < \pi$, and $\cos B = -\frac{5}{13}$, $\pi < B < \frac{3\pi}{2}$. Find each of the following:

- $\sin(A + B)$,
- $\tan(A + B)$,
- the quadrant in which $A + B$ lies.

8. Use identities to find the **exact values** of the following, without using calculators:

- $\cos 25^\circ \cos 125^\circ - \sin 25^\circ \sin 125^\circ$,
- $\sin \frac{5\pi}{12}$,
- $\tan \frac{\pi}{12}$.

9. Verify that

$$\frac{2 \cos 2x}{\sin 2x} = \cot x - \tan x.$$

10. Find the exact value of y in each of the following without using a calculator

- $y = \arcsin\left(\frac{\sqrt{3}}{2}\right)$
- $y = \sin\left(\cos^{-1}\left(\frac{8}{17}\right)\right)$

11. Solve each equation for exact solutions of angles over the interval $[0, 2\pi]$

- $2 \sin^2 \alpha + 3 \sin \alpha + 1 = 0$
- $2 \cos^2 \theta - 2 \sin^2 \theta + 1 = 0$

12. Let $\mathbf{u} = \langle 6, -3 \rangle$ and $\mathbf{v} = \langle -14, 8 \rangle$. Find the following

- $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$,
- $5\mathbf{u} + 2\mathbf{v}$,
- $\mathbf{u} \cdot \mathbf{v}$.