1. John wants to measure the height of a flagpole. From a point 23.5 ft from the base of the flagpole, he finds that the angle of elevation to the top of the flagpole is 35°. What is the height of the flagpole? (Give the answer accurate to 2 decimals.)

![Diagram of a right triangle with angle 35° and base 23.5 ft]

2. Without using a calculator, find a solution for $\theta$ for each of the following equations:
   a) $\cos \theta = \sin(2\theta - 30°)$,
   b) $\sec(3\theta + 10°) = \csc(\theta + 8°)$.

3. A wheel is rotating at 10 radians per sec, and the wheel has a 20-inch diameter.
   a) Through how many radians does the wheel rotate in 1 minute?
   b) What is the angular speed of the wheel in radians per minute?
   c) What is the speed of a point on the rim in inches per minute?

4. Consider the following function
   \[
   y = 16.5 \sin \left[ \frac{\pi}{6} (x - 4) \right] + 67.5
   \]  
   (1)
   a) What is the vertical displacement of $y$?
   b) What is the amplitude of $y$?
   c) What is the period of $y$?
   d) What is the horizontal displacement (Phase Shift) of $y$?

5. In the following graph, the thick line is the $x$-axis and the $x$-axis is enumerated in degrees.
   a) What is the period of the function?
   b) What is the amplitude of the function?
   c) What is the vertical displacement of the function?
d) What is the horizontal displacement of the function?
e) Write down the equation of the function displaced above

6. Verify the identity:

\[(1 + \tan x)^2 + (1 - \tan x)^2 = \frac{1}{1 - \sin^2 x} + \frac{1}{\cos^2 x}.\]

7. If \(\theta\) is in quadrant II and \(\sin \theta = \frac{3}{5}\), find each exact values without using a calculator of:
   a) \(\sin \left(\theta + \frac{2\pi}{3}\right)\);
   b) \(\tan(2\theta)\).

8. Find the exact values of the following without using a calculator:
   a) \(2 \sin 22.5^\circ \cos 22.5^\circ\).
   b) \(\frac{\tan 100^\circ + \tan 20^\circ}{1 - \tan 100^\circ \tan 20^\circ}\).

9. Find the exact value of \(y\) in each of the following without using a calculator
   a) \(y = \arcsin\left(-\frac{\sqrt{2}}{2}\right)\),
   b) \(y = \cos(2 \arcsin(\frac{12}{13}))\).

10. Solve each equation for exact solutions over the interval \([0, 2\pi)\).
    a) \(4 \sin x \cos x = 1\),
    b) \(\sin x + \cos x = 0\).

11. Find the remaining angles and sides of triangle \(ABC\) if it is given that \(A = 30^\circ\), \(B = 20^\circ\) and \(a = 45\). (Give the answer accurate to 2 decimals.)

12. How many triangles \(ABC\) are possible if \(a = 15, b = 25, c = 80^\circ\). Justify your answer.
\[
\begin{align*}
\sin (A + B) &= \sin A \cos B + \sin B \cos A \\
\cos (A + B) &= \cos A \cos B - \sin A \sin B \\
\tan (A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\
\tan (A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B}
\end{align*}
\]

\[
\begin{align*}
\sin C + \sin D &= 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2} \\
\sin C - \sin D &= 2 \cos \frac{C + D}{2} \sin \frac{C - D}{2} \\
\cos C + \cos D &= 2 \cos \frac{C + D}{2} \cos \frac{C - D}{2} \\
\cos C - \cos D &= 2 \sin \frac{C + D}{2} \sin \frac{D - C}{2}
\end{align*}
\]

\[
\begin{align*}
2 \sin A \cos B &= \sin (A + B) + \sin (A - B) \\
2 \cos A \sin B &= \sin (A + B) - \sin (A - B) \\
2 \cos A \cos B &= \cos (A + B) + \cos (A - B) \\
2 \sin A \sin B &= \cos (A - B) - \cos (A + B)
\end{align*}
\]

\[
\begin{align*}
\sin 2\theta &= 2 \sin \theta \cos \theta \\
\cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\
&= 2 \cos^2 \theta - 1 \\
&= 1 - 2 \sin^2 \theta \\
\tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta}
\end{align*}
\]

\[
\begin{align*}
\sin \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{2}} \\
\cos \frac{\theta}{2} &= \pm \sqrt{\frac{1 + \cos \theta}{2}} \\
\tan \frac{\theta}{2} &= \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin x}
\end{align*}
\]

Law of the Sines

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R
\]
where, $R$ is the radius of the triangle’s circumcircle.

**Law of the Cosines**

\[
\begin{align*}
  a^2 &= b^2 + c^2 - 2bc \cos A \\
  b^2 &= c^2 + a^2 - 2ca \cos B \\
  c^2 &= a^2 + b^2 - 2ab \cos C
\end{align*}
\]

**Area of a Triangle**

\[
\begin{align*}
  A &= \frac{1}{2} bc \sin A \\
  A &= \frac{1}{2} ca \sin B \\
  A &= \frac{1}{2} ab \sin C
\end{align*}
\]

**Heron’s Area Formula**

\[
A = \sqrt{s(s - a)(s - b)(s - c)}
\]

where, \( s = \frac{1}{2}(a + b + c) \).
Solution key for Math 1321, Spring 2016

1. We have

\[ h = 23.5 \times \tan 35^\circ = 16.45 \text{ ft.} \]

2. Make the angles on the left and those on the right complementary. For
a) set \( \theta = 90^\circ - 2\theta + 30^\circ \), so \( \theta = 40^\circ \). For b) set \( 3\theta + 10^\circ = 90^\circ - \theta - 8^\circ \).
Then \( 4\theta = 72^\circ \), so \( \theta = 24^\circ \).

3. a) \( 10 \times 60 = 600 \) radians, b) \( 600 \) rad/min, c) The radius is 10 inches.
Multiply this by the angular speed to get \( 10 \times 600 = 6000 \) in/min.

4. a) The vertical displacement is 67.5.
   b) The amplitude is 16.5.
   c) The period is \( 2\pi/(\pi/6) = 12 \).
   d) The horizontal displacement is 4 to the right.

5. a) The period is \( 180^\circ \).
   b) The amplitude is 9.
   c) The vertical displacement is 5.
   d) The horizontal displacement is \( 300^\circ \) to the right.
   e) \( y = 9 \sin(2(x - 300^\circ)) + 5 \).
Note: This problem might have more than 1 correct answer!

6. We have

\[
\begin{align*}
(1 + \tan x)^2 + (1 - \tan x)^2 \\
= (1 + 2\tan x + \tan^2 x) + (1 - 2\tan x + \tan^2 x) \\
= 2 + 2\tan^2 x \\
= 2(1 + \tan^2 x) \\
= 2(\sec^2 x) \quad \text{(by pythagorean identity} \ 1 + \tan^2 x = \sec^2 x) \\
= 2/\cos^2 x \quad \text{(by reciprocal identity} \ \sec x = 1/\cos x) \\
= 1/\cos^2 x + 1/\cos^2 x \\
= 1/(1 - \sin^2 x) + 1/\cos^2 x \quad \text{(by the pythagorean identity} 1 - \sin^2 x = \cos^2 x)
\end{align*}
\]
7. We have

\[
\cos \theta = -\sqrt{1 - \sin^2 \theta} = -\sqrt{(1 - (3/5)^2} = -4/5
\]

\[
\tan \theta = \sin \theta / \cos \theta = (3/5) / (-4/5) = -3/4.
\]

so

\[
\sin \left( \theta + \frac{2\pi}{3} \right) = \sin \theta \cos \frac{2\pi}{3} + \cos \theta \sin \frac{2\pi}{3}
\]

\[
= \left[ \frac{3}{5} \times (-\frac{1}{2}) \right] + \left[ (-\frac{4}{5}) \times \frac{\sqrt{3}}{2} \right] = -\left( \frac{3 + 4\sqrt{3}}{10} \right).
\]

For b) we have

\[
\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \times (-3/4)}{1 - (-3/4)^2} = \frac{-6/4}{1 - 9/16} = \frac{-6/4}{7/16} = \frac{-24}{7}.
\]

8. a) \(2 \sin 22.5^\circ \cos 22.5^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}\).

b) We have

\[
\tan 100^\circ + \tan 20^\circ = \tan 120^\circ = -\sqrt{3}.
\]

9. a) \(y = -\frac{\pi}{4}\),

b) We have

\[
\cos(2 \arcsin \frac{12}{13}) = 1 - 2 \sin^2(\arcsin \frac{12}{13})
\]

\[
= 1 - 2 \times \frac{144}{169} = \frac{-119}{169}.
\]

10. a) Using the double-angle formula we transform this into \(\sin 2x = 1/2\).

So \(2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}\). We obtain \(x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}\).

b) Write this as \(\tan x = -1\). So \(x = \frac{3\pi}{4}, \frac{7\pi}{4}\).

11. \(C = 180^\circ - A - B = 130^\circ\). By the law of sines

\[
\frac{45}{\sin 30^\circ} = \frac{b}{\sin 20^\circ} = \frac{c}{\sin 130^\circ}.
\]

So

\[
b = \frac{45 \sin 20^\circ}{\sin 30^\circ} \approx 30.78 \quad c = \frac{45 \sin 130^\circ}{\sin 30^\circ} \approx 68.94.
\]
12. We have

\[ \frac{15}{\sin 80^\circ} = \frac{25}{\sin B} \, . \]

So

\[ \sin B = \frac{25 \sin 80^\circ}{15} \approx 1.641 \, . \]

So no such triangle exists.