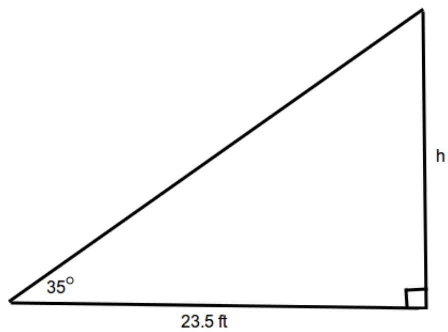
MATH 1321 - FINAL EXAMINATION
Spring 2016

SHOW ALL YOUR WORK. EACH PROBLEM IS WORTH THE SAME NUMBER OF POINTS.

1. John wants to measure the height of a flagpole. From from a point 23.5 ft from the base of the flagpole, he finds that the angle of elevation to the top of the flagpole is $35°$. What is the height of the flagpole? (Give the answer accurate to 2 decimals.)



2. Without using a calculator, find a solution for θ for each of the following equations:
   a) $\cos\theta = \sin(2\theta - 30°)$,
   b) $\sec(3\theta + 10°) = \csc(\theta + 8°)$.

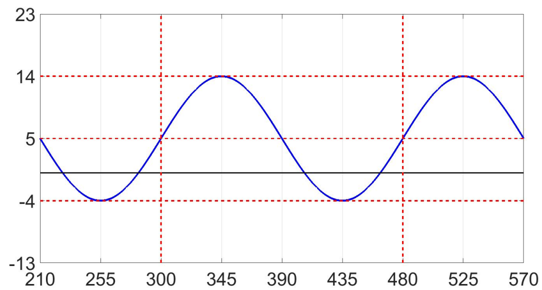
3. A wheel is rotating at 10 radians per sec, and the wheel has a 20-inch diameter.
   a) Through how many radians does the wheel rotate in 1 minute?
   b) What is the angular speed of the wheel in radians per minute?
   c) What is the speed of a point on the rim in inches per minute?

4. Consider the following function

$$y = 16.5 \sin\left[\frac{\pi}{6}(x - 4)\right] + 67.5 \tag{1}$$

   a) What is the vertical displacement of y?
   b) What is the amplitude of y?
   c) What is the period of y?
   d) What is the horizontal displacement (Phase Shift) of y?

5. In the following graph, the thick line is the x-axis and the x-axis is enumerated in degrees.
   a) What is the period of the function?
   b) What is the amplitude of the function?
   c) What is the vertical displacement of the function?



d) What is the horizontal displacement of the function?

e) Write down the equation of the function displaced above

6. Verify the identity:

$$(1+\tan x)^2+(1-\tan x)^2=\frac{1}{1-\sin^2 x}+\frac{1}{\cos^2 x}.$$

7. If θ is in quadrant II and $\sin\theta=\frac{3}{5}$, find each exact values without using a calculator of:
a) $\sin\left(\theta+\frac{2\pi}{3}\right)$;
b) $\tan(2\theta)$.

8. Find the exact values of the following without using a calculator:

$$\text{a) } 2\sin 22.5^\circ \cos 22.5^\circ.$$

$$\text{b) } \frac{\tan 100^\circ+\tan 20^\circ}{1-\tan 100^\circ \tan 20^\circ}.$$

9. Find the exact value of y in each of the following without using a calculator
a) $y=\arcsin(-\frac{\sqrt{2}}{2})$,
b) $y=\cos(2\arcsin(\frac{12}{13}))$.

10. Solve each equation for exact solutions over the interval $[0,2\pi)$.

a) $4\sin x\cos x=1$,
b) $\sin x+\cos x=0$.

11. Find the remaining angles and sides of triangle ABC if it is given that $A=30^\circ$, $B=20^\circ$ and $a=45$. (Give the answer accurate to 2 decimals.)

12. How many triangles ABC are possible if $a=15$, $b=25$, $c=80^\circ$. Justify your answer.

FORMULA SHEET

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$$

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$
$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$
$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$
$$\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$
$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$
$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$
$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin x}$$

Law of the Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin B} = 2R$$

where, R is the radius of the triangle's circumcircle.

Law of the Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = c^2 + a^2 - 2ca\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a Triangle

$$\text{A} = \frac{1}{2}bc\sin A$$
$$\text{A} = \frac{1}{2}ca\sin B$$
$$\text{A} = \frac{1}{2}ab\sin C$$

Heron's Area Formula

$$\text{A} = \sqrt{s(s-a)(s-b)(s-c)}$$

where, $s = \frac{1}{2}(a+b+c)$.