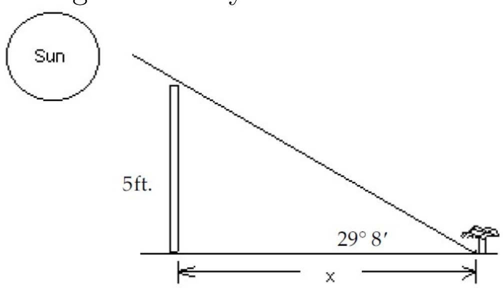
MATH 1321 - FINAL EXAMINATION
Fall 2017

SHOW ALL YOUR WORK. EACH PROBLEM IS WORTH THE SAME NUMBER OF POINTS.

1. In one area, the lowest angle of elevation of the sun in winter is $29°8'$. Find the minimum distance x that a plant needing full sun can be placed from a fence that is 5 feet high. Round your answer to the tenths place when necessary.



2. Find all values of θ, if θ is in the interval $[0, 360°)$ and $\sin\theta = \frac{\sqrt{3}}{2}$.

3. A point P on the edge of a wheel is rotating 35 times per minute. The diameter of the wheel is 3.0 meters. You may leave π in your answers.
(a) Find the angular speed of P in radians per second.
(b) Find the linear speed of P in meters per second.

4. Solve each equation for exact solutions over the interval $[0, 2\pi)$.
(a) $\cos^2 x + 2\cos x + 1 = 0$
(b) $\sin 2x = -\sin x$.

5. Given is the function
$$y = 3 - \sin\left(2x - \frac{\pi}{2}\right).$$
Find the amplitude, the period, any vertical translation, and any phase shift of the graph of the function. What is the range of the function?

6. Given that $\sin(\alpha) = \frac{3}{5}$, $\cos(\beta) = -\frac{5}{13}$, and α is in the quadrant II and β is in the quadrant III. Find $\sin(\alpha + \beta)$ and $\cos(\alpha - \beta)$.

7. Find the exact value of $\sin 15°$.

8. Given that $\cos\theta = -\frac{4}{5}$, and $\frac{\pi}{2} < \theta < \pi$, find $\cos 2\theta$ and $\sin\frac{\theta}{2}$.

9. Find the exact value of y in each of the following without using a calculator:
(a) $y = \arctan(-1)$,
(b) $y = \cos\left[\arcsin\left(\frac{1}{4}\right)\right]$
(c) $y = \sec\left[2\arcsin\left(-\frac{1}{3}\right)\right]$.

10. Write the following trigonometric expression as an algebraic expression in u, $u > 0$. $\sin\left[\tan^{-1}\left(\frac{u}{5}\right)\right]$.

11. Solve triangle ABC if $B = 29.5°$, $b = 19.45$, $c = 19.75$. If necessary, round angles to the nearest tenth and side lengths to the nearest hundredth.

12. Two airplanes leave an airport at the same time, one going northwest (at 45° angle with the NS axis) at 414 mph and the other going east at 343 mph. How far apart are the planes after 4 hours (to the nearest mile)?

FORMULA SHEET

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

$$\begin{aligned}
\sin C + \sin D &= 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\\
\sin C - \sin D &= 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}\\
\cos C + \cos D &= 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}\\
\cos C - \cos D &= 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}
\end{aligned}$$

$$\begin{aligned}
2\sin A\cos B &= \sin(A+B) + \sin(A-B)\\
2\cos A\sin B &= \sin(A+B) - \sin(A-B)\\
2\cos A\cos B &= \cos(A+B) + \cos(A-B)\\
2\sin A\sin B &= \cos(A-B) - \cos(A+B)
\end{aligned}$$

$$\begin{aligned}
\sin 2\theta &= 2\sin\theta\cos\theta\\
\cos 2\theta &= \cos^2\theta - \sin^2\theta\\
&= 2\cos^2\theta - 1\\
&= 1 - 2\sin^2\theta\\
\tan 2\theta &= \frac{2\tan\theta}{1-\tan^2\theta}
\end{aligned}$$

$$\begin{aligned}
\sin\frac{\theta}{2} &= \pm\sqrt{\frac{1-\cos\theta}{2}}\\
\cos\frac{\theta}{2} &= \pm\sqrt{\frac{1+\cos\theta}{2}}\\
\tan\frac{\theta}{2} &= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin x}
\end{aligned}$$

Law of the Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin B} = 2R$$

where, R is the radius of the triangle's circumcircle.

Law of the Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = c^2 + a^2 - 2ca\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a Triangle

$$\text{A} = \frac{1}{2}bc\sin A$$
$$\text{A} = \frac{1}{2}ca\sin B$$
$$\text{A} = \frac{1}{2}ab\sin C$$

Heron's Area Formula

$$\text{A} = \sqrt{s(s-a)(s-b)(s-c)}$$

where, $s = \frac{1}{2}(a + b + c)$.