SHOW ALL YOUR WORK. EACH PROBLEM IS WORTH THE SAME NUMBER OF POINTS.

1. In one area, the lowest angle of elevation of the sun in winter is $29^\circ 8'$. Find the minimum distance $x$ that a plant needing full sun can be placed from a fence that is 5 feet high. Round your answer to the tenths place when necessary.

2. Find all values of $\theta$, if $\theta$ is in the interval $[0, 360^\circ)$ and $\sin \theta = \frac{\sqrt{3}}{2}$.

3. A point $P$ on the edge of a wheel is rotating 35 times per minute. The diameter of the wheel is 3.0 meters. You may leave $\pi$ in your answers.
   (a) Find the angular speed of $P$ in radians per second.
   (b) Find the linear speed of $P$ in meters per second.

4. Solve each equation for exact solutions over the interval $[0, 2\pi)$.
   (a) $\cos^2 x + 2 \cos x + 1 = 0$
   (b) $\sin 2x = -\sin x$.

5. Given is the function
   
   $$y = 3 - \sin \left(2x - \frac{\pi}{2}\right).$$

   Find the amplitude, the period, any vertical translation, and any phase shift of the graph of the function. What is the range of the function?

6. Given that $\sin(\alpha) = \frac{3}{5}$, $\cos(\beta) = -\frac{5}{17}$, and $\alpha$ is in the quadrant II and $\beta$ is in the quadrant III. Find $\sin(\alpha + \beta)$ and $\cos(\alpha - \beta)$.

7. Find the exact value of $\sin 15^\circ$.

8. Given that $\cos \theta = -\frac{4}{5}$, and $\frac{\pi}{2} < \theta < \pi$, find $\cos 2\theta$ and $\sin \frac{\theta}{2}$.

9. Find the exact value of $y$ in each of the following without using a calculator:
   (a) $y = \arctan(-1)$,
   (b) $y = \cos \left[\arcsin \left(\frac{1}{3}\right)\right]$,
   (c) $y = \sec \left[2 \arcsin \left(-\frac{1}{3}\right)\right]$.
10. Write the following trigonometric expression as an algebraic expression in $u$, $u > 0$.
\[ \sin \left[ \tan^{-1} \left( \frac{u}{5} \right) \right]. \]

11. Solve triangle $ABC$ if $B = 29.5^\circ$, $b = 19.45$, $c = 19.75$. If necessary, round angles to the nearest tenth and side lengths to the nearest hundredth.

12. Two airplanes leave an airport at the same time, one going northwest (at $45^\circ$ angle with the NS axis) at 414 mph and the other going east at 343 mph. How far apart are the planes after 4 hours (to the nearest mile)?
FORMULA SHEET

\[\sin (A + B) = \sin A \cos B + \sin B \cos A\]
\[\sin (A - B) = \sin A \cos B - \cos A \sin B\]
\[\cos (A + B) = \cos A \cos B - \sin A \sin B\]
\[\cos (A - B) = \cos A \cos B + \sin A \sin B\]
\[\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\]
\[\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}\]

\[\sin C + \sin D = 2 \sin \left( \frac{C + D}{2} \right) \cos \left( \frac{C - D}{2} \right)\]
\[\sin C - \sin D = 2 \cos \left( \frac{C + D}{2} \right) \sin \left( \frac{C - D}{2} \right)\]
\[\cos C + \cos D = 2 \cos \left( \frac{C + D}{2} \right) \cos \left( \frac{C - D}{2} \right)\]
\[\cos C - \cos D = 2 \sin \left( \frac{C + D}{2} \right) \sin \left( \frac{D - C}{2} \right)\]

\[2 \sin A \cos B = \sin (A + B) + \sin (A - B)\]
\[2 \cos A \sin B = \sin (A + B) - \sin (A - B)\]
\[2 \cos A \cos B = \cos (A + B) + \cos (A - B)\]
\[2 \sin A \sin B = \cos (A - B) - \cos (A + B)\]

\[\sin 2\theta = 2 \sin \theta \cos \theta\]
\[\cos 2\theta = \cos^2 \theta - \sin^2 \theta\]
\[= 2 \cos^2 \theta - 1\]
\[= 1 - 2 \sin^2 \theta\]
\[\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}\]

\[\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}\]
\[\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}\]
\[\tan \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin x}\]

Law of the Sines

\[\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\]
where, $R$ is the radius of the triangle’s circumcircle.

Law of the Cosines

\[
\begin{align*}
a^2 &= b^2 + c^2 - 2bc \cos A \\
b^2 &= c^2 + a^2 - 2ca \cos B \\
c^2 &= a^2 + b^2 - 2ab \cos C
\end{align*}
\]

Area of a Triangle

\[
\begin{align*}
A &= \frac{1}{2}bc \sin A \\
A &= \frac{1}{2}ca \sin B \\
A &= \frac{1}{2}ab \sin C
\end{align*}
\]

Heron’s Area Formula

\[
A = \sqrt{s(s-a)(s-b)(s-c)}
\]

where, $s = \frac{1}{2}(a + b + c)$. 