1. Find $h$ as indicated in the figure below. Round the nearest foot.

2. Find exact values of $\cos(\theta)$, $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$, $\csc(\theta)$ if $\sin(\theta) = \frac{3}{5}$ and $\theta$ is in the second quadrant.

3. A bicycle with a 26-inch wheel (diameter) travels 200 feet. How many revolutions does the wheel make (to the nearest revolution)?

4. The function graphed is of the form $y = a\sin(bx)$ or $y = a\cos(bx)$, where $b > 0$. Determine the equation of the graph below.

5. Use trigonometry identities to find the exact value of $\cos(-75^\circ)$.

6. Write $\cot(x)$ on terms of $\sin(x)$ for an angle $x$ in the third quadrant.
7. If \( \theta \) is in quadrant II and \( \sin(\theta) = \frac{2}{3} \), find each exact value without using a calculator of:

a) \( \cos(\theta + \frac{3\pi}{4}) \),  
b) \( \sin(\theta - \frac{\pi}{6}) \).

8. Find the exact values of the following without using a calculator:

a) \( \sin 15^\circ \cos 15^\circ \),  
b) \( \frac{2\tan(22.5^\circ)}{1 - \tan^2(22.5^\circ)} \).

9. Find the exact value of \( y \) in the following without using the calculator

\[
y = \cos\left(2\arcsin\frac{4}{5}\right).
\]

10. Solve each equation for solutions in the interval \([0, 2\pi)\):

a) \( \sin x \cos x = 1 \),  
b) \( \sin\frac{x}{2} + \cos\frac{x}{2} = 0 \).

11. Find the remaining angles and sides of triangles \( \triangle ABC \) if it is given that \( A=20^\circ \), \( B=50^\circ \) and \( b=12 \).

(Give the answer accurate to 2 decimals.)

12. How many triangles \( \triangle ABC \) are possible if \( a= 6 \), \( c= 9 \) and \( B= 70^\circ \)? Justify your answer.
Math 1321 Final Exam Spring 2017 Solutions

1. Denote the other leg in the small right triangle by $x$. Then in the small right triangle $x/h = \cot 56.5^\circ$. Thus $x = h \cot 56.5^\circ$. In the large right triangle, $h/(x + 131) = \tan 24.2^\circ$. Thus

$$h = h \cot 56.5^\circ \tan 24.2^\circ + 131 \tan 24.2^\circ,$$

so

$$h = \frac{131 \times \tan 24.2^\circ}{1 - \tan 24.2^\circ \cot 56.5^\circ} \approx 83.668.$$

Approximating to the nearest foot we obtain the answer 84 feet.

2. We have

$$\cos \theta = -\sqrt{1 - \sin^2 \theta} = -\sqrt{1 - \frac{3^2}{5^2}} = -\frac{4}{5},$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = -\frac{3}{4}, \quad \cot \theta = \frac{1}{\tan \theta} = -\frac{4}{3},$$

$$\sec \theta = \frac{1}{\cos \theta} = -\frac{5}{4}, \quad \csc \theta = \frac{1}{\sin \theta} = \frac{5}{3}.$$

3. 200 feet is $200 \times 12 = 2400$ inches. The length of the circumference is $26 \times \pi \approx 81.68$. The number of revolutions is $2400/81.68 \approx 29.3$. That is approximately 29 revolutions.

4. The graph is $5 \sin(x/3)$.

5. There are many possible approaches. Here is one:

$$\cos(-75^\circ) = \cos 75^\circ = \cos(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}.$$

6. Since $x$ is in the third quadrant, both $\sin x$ and $\cos x$ are negative. We have

$$\cot x = \frac{\cos x}{\sin x} = -\frac{\sqrt{1 - \sin^2 x}}{\sin x} = -\frac{\sqrt{1 - \sin^2 x}}{\sin x}.$$
7. (a) We have \( \cos \theta = -\sqrt{1 - \sin^2 \theta} = -\frac{\sqrt{5}}{3} \). Then

\[
\cos \left( \theta + \frac{3\pi}{4} \right) = \cos \theta \cos \frac{3\pi}{4} - \sin \theta \sin \frac{3\pi}{4} = \frac{\sqrt{10}}{6} - \frac{2\sqrt{2}}{6} = \frac{\sqrt{10} - 2\sqrt{2}}{6}.
\]

(b) Also,

\[
\sin \left( \theta - \frac{\pi}{6} \right) = \sin \theta \cos \frac{\pi}{6} - \cos \theta \sin \frac{\pi}{6} = \frac{2\sqrt{3} - \sqrt{5}}{6}.
\]

8. (a) We have

\[
\sin 15^\circ \cos 15^\circ = \frac{1}{2} \sin 30^\circ = \frac{1}{4}.
\]

(b) The expression is equal to \( \tan 45^\circ = 1 \).

9. We have

\[
\cos \left( 2 \arcsin \frac{4}{5} \right) = 2\cos^2 \arcsin \frac{4}{5} - 1 = 2 \left( \frac{3}{5} \right)^2 - 1 = -\frac{7}{25}.
\]

10. (a) The equation can be written \( \sin 2x = 1/2 \). This means that \( x = \pi/12, x = 5\pi/12, x = 13\pi/12, x = 17\pi/12 \).

(b) Write it as \( \tan \frac{x}{2} = -1 \). Then \( x = 3\pi/2 \).

11. \( C = 180^\circ - 50^\circ - 20^\circ = 110^\circ \). Then by the law of sines \( a = 5.36 \) and \( c = 14.72 \).

12. Using the law of cosines we have \( b^2 = a^2 + c^2 - 2ac \cos B \). We see that side \( b \) can be computed uniquely and thus there is only one such triangle. Note that \( a^2 + c^2 - 2ac \cos B > a^2 + c^2 - 2ac \). So \( b \) exists.