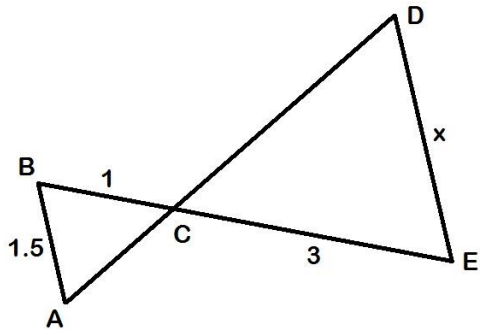
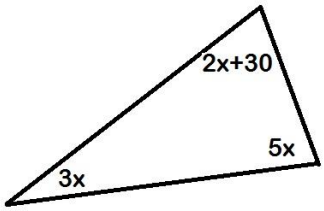
FINAL EXAM
TRIGONOMETRY
MATH 1321
TUESDAY MAY 15TH

There are twenty questions. Each question is worth five points. **Calculators are not permitted**, therefore provide exact solutions to problems and not approximations.

(1) In the triangles given on the picture, sides AB and DE are parallel. The sides BC, BA and CE have lengths 1, 1.5 and 3 respectively. Find the length of the side DE.



(2) Find the measures of each angle of the triangle in degrees



(3) Find all possible values of θ in radians, if θ is in the interval $[0, 2\pi)$, and has the given function value

$$\sin\theta = -\frac{1}{2}.$$

(4) A ship travels 30km on a bearing of 5° and then travels on a bearing 95° for 40km. How far is the ship from the starting point?

(5) (a) Convert the following degree measures to radian measures:
  (i) $150° = ?$
  (ii) $-315° = ?$
  (b) Convert the following radian measure to degree measures:
  (i) $-\frac{2\pi}{3} = ?$
  (ii) $\frac{9\pi}{4} = ?$

(6) Find exact function values for:
  (a) $\sin\frac{4\pi}{3} = ?$
  (b) $\tan\frac{5\pi}{6} = ?$

(7) Two gears with radii 4cm and 12cm are adjusted so that the smaller gear drives the larger one. If the smaller gear rotates through an angle of 270°, through how many degrees does the larger gear rotate?

(8) Determine the periods of the following functions:
   (a) $y = 2\cos(\frac{x}{4})$
   (b) $y = 3\tan(2x)$

(9) (a) The graph of the tangent function intersects the x-axis for all numbers of what form?
   (b) What is the smallest positive number x for which $\cot x = 0$?

(10) Sketch the graph of the function $y = -\frac{1}{2}\cos(2x)$ over the interval $[0, p]$ where p is the period of this function. Indicate the maximum and minimum values of the function and where the function crosses the x-axis.

(11) Write the given expression in terms of sin and cosine, and then simplify the expression.

$$\tan^2\theta(\sin^2\theta - 1).$$

(12) Find the exact value of $\cos 15°$.

(13) Verify the identity,

$$4\sin^2 x\cos^2 x + \cos^2(2x) = 1.$$

(14) Solve the equation,

$$\cos x(2\sin x + \sqrt{3}) = 0,$$

for x in the interval $[0, 2\pi)$.

(15) Evaluate the expression,

$$\cos(\arctan 1 + \arcsin 1).$$

(16) For the vectors $\overrightarrow{\mathbf{u}} = \langle 2, 1\rangle$ and $\overrightarrow{\mathbf{v}} = \langle -3, 1\rangle$:
   (a) Evaluate $-2\overrightarrow{\mathbf{u}} + 4\overrightarrow{\mathbf{v}}$.
   (b) Find the angle between $\overrightarrow{\mathbf{u}}$ and $\overrightarrow{\mathbf{v}}$.

(17) If the lengths of the three sides of a given triangle are 3in, 6in and 7in. Find the area of this triangle.

(18) In the triangle ABC, the sides a, b and c are opposite to the angles A, B and C respectively. Given that $A = 60°$, $B = 75°$, $c = \sqrt{2}$cm and $\sin 75° = \frac{1+\sqrt{3}}{2\sqrt{2}}$, find the area of the triangle.

(19) Perform the following operations and write the answer in standard form.
   (a) $(4 - i)(5 + 2i) =?$
   (b) $\frac{2-5i}{1+i} =?$

(20) Write the following complex numbers in trigonometric (polar) form, where the angle $\theta \in [0, 2\pi)$.
   (a) $1 + i =?$
   (b) $1 - i\sqrt{3} =?$