MATH 1330 – Final Exam, Form A
Spring 2014

NAME__________________________________
Section _______

Please show all work neatly.

On each of the problems you must show the correct formula with correct numbers and appropriate units substituted into those formulas for a majority of the points. On the Venn diagram problem and the Tree diagram problem, you will receive the majority of the points for labeling each diagram correctly.

(6 pts) 1. The weekly demand and supply functions for a kind of badminton racket, Li-Ning Woods N90 II (as used by Lin Dan, a professional badminton men's singles player from China, two-time Olympic Champion, five-time World Champion, and five-time All England Champion) are given by

$$p = d(x) = -x^2 + 2x + 233$$

$$p = s(x) = 2x^2 + 2x + 206$$

respectively, where p is measured in dollars and x is measured in units of a thousand.

What is the **market equilibrium point**? Hint: (equilibrium quantity, equilibrium price).

(6 pts) 2. The estimated daily cost, in dollars, by a company for manufacturing and selling x units is

$$C(x) = 4x^2 - 400x + 15000$$

a. How many units should be produced and sold for the company to minimize the daily cost?

b. What is the minimum daily cost realizable?

(6 pts) 3. A radioactive substance decays according to the exponential growth/decay mode with a half-life of 27 days. If 200 g of this substance are present initially

a. What is the decay constant to five decimal places?

b. What amount will be left after 126 days? (Round your answer to two decimal places.)

(5 pts) 4. The management of Gibraltar Brokerage Services anticipates a capital expenditure of \$20,000 in 3 years for the purchase of new computers and has decided to set up a sinking fund to finance this purchase. If the fund earns interest at the rate of 10% per year compounded quarterly, determine the size of each (equal) quarterly installment that should be deposited in the fund.

(6 pts) 5. The price of a new car is \$16,000. Assume that an individual makes a down payment of 25% toward the purchase of the car and secures financing for the balance at the rate of 10% per year compounded monthly.

a. What monthly payment will they be required to make if the car is financed over a period of 36 months?

b. How much interest will they pay?

(5 pts) 6. If Dan deposits money into an account that earns 5.5% per year compounded continuously, how long will it take for his investment to be doubled? (Round your answer to one decimal place.)

(8 pts) 7.

a. Ben plans to retire in 40 years and wishes to withdraw $2,000 per month for 20 years from his retirement account at that time. How much will Ben need in the account at the time of his retirement if the account earns interest at 7%/year compounded monthly?

b. How much must he invest monthly for 40 years, earning interest at 7%/year compounded monthly, to have the required amount at the time of his retirement?

(12 pts) 8. The Smith's have decided to purchase a home for $245,000. The Smith's will make a down payment of $25,000. They will then secure a mortgage with interest charged at a rate of 7%/year on the unpaid balance. The loan is to be amortized over 15 years.

a. What monthly payment will the Smith's be required to make?

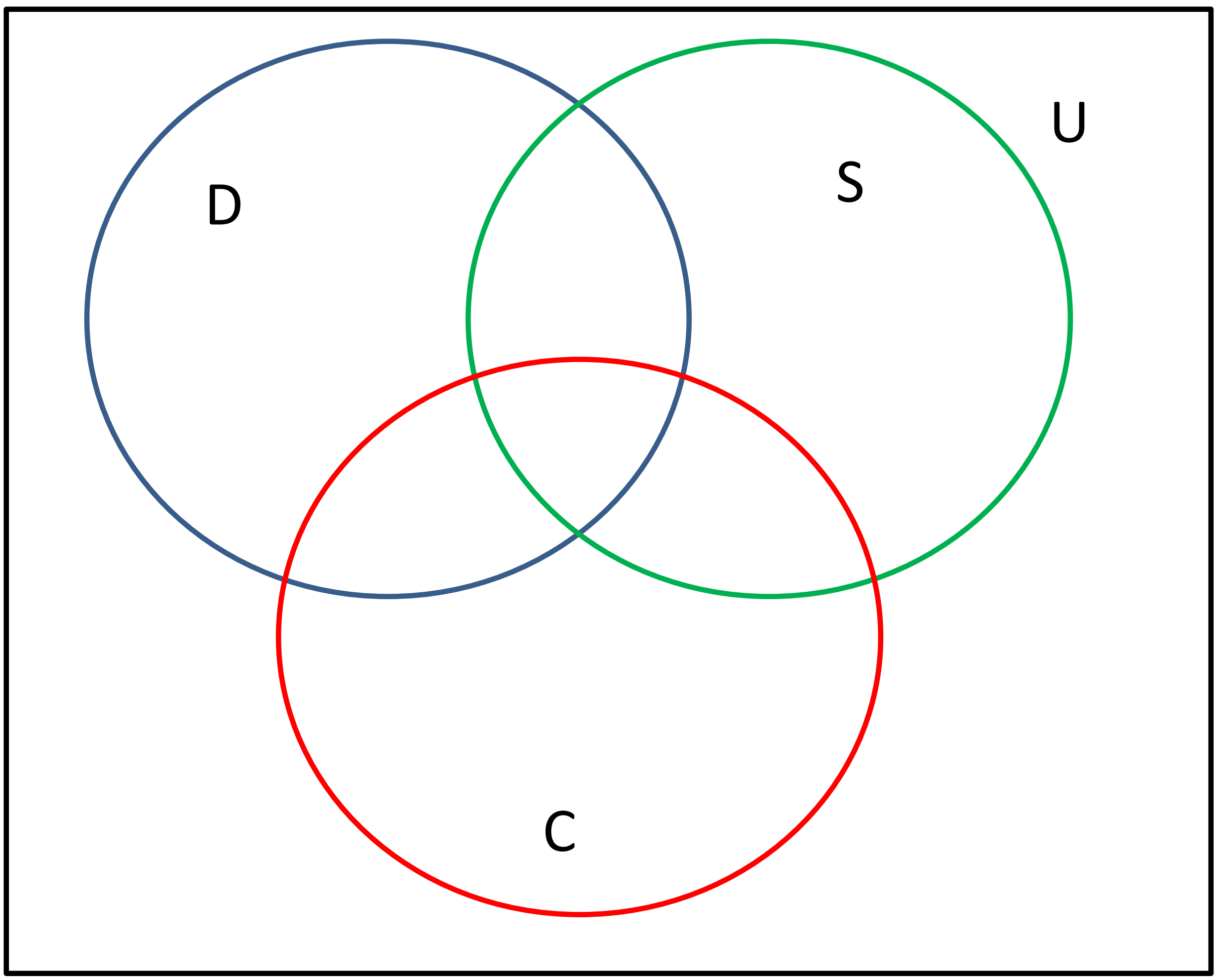
b. After 5 years, what is the current outstanding principal?

c. What is their equity after 5 years (disregarding appreciation)?

d. How much interest will they pay the mortgage company?

(6 pts) 9. 300 NFL fans were asked about their preferences towards the Denver Broncos (D), the Seattle Seahawks (S), and the Carolina Panthers (C). The results are as follows:

148 fans said they liked the Denver Broncos
102 fans said they liked the Seattle Seahawks
75 fans said they liked the Carolina Panthers
15 fans said they liked the Denver Broncos and the Seattle Seahawks
17 fans said they liked Seattle Seahawks and the Carolina Panthers
38 fans said they liked the Denver Broncos and the Carolina Panthers
5 fans liked all three teams



Out of the 300 fans surveyed, how many fans liked

a. None of the teams?

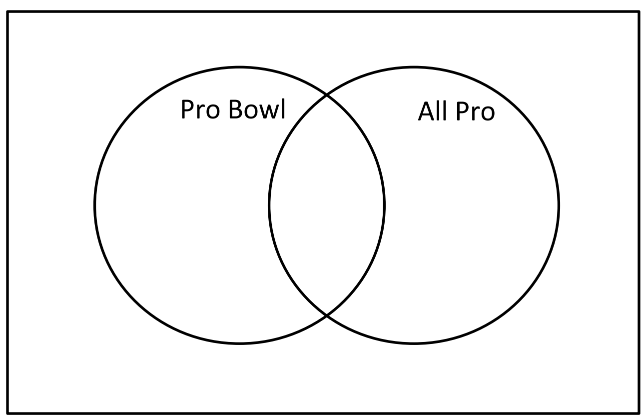
b. Exactly one of the three teams?

c. Only the Seattle Seahawks and the Carolina Panthers?

(4 pts) 10. A Lubbock restaurant offers a daily 3-course lunch special where customers can select an appetizer, an entrée, and a dessert from a choice of 3 appetizers, 8 entrees, and 3 desserts. In how many different ways can complete lunches be ordered from the daily lunch special?

(6 pts) 11. A freshman at Texas Tech is deciding what classes she should enroll in for the fall semester. She must select two of seven business courses, one of two math courses, one of three economics courses, and either one of three history courses or one of six elective courses. How many different ways can the freshman enroll in classes for the following fall semester?

(4 pts) 12. Kam Chancellor is being considered for the Pro-Bowl team in Hawaii and also for the All-Pro team. The probability that Kam Chancellor will be placed on the Pro-Bowl team is 0.75. The probability that he will be placed on the All-Pro team is 0.6, and the probability that he will be placed on both teams is 0.5.



a. What is the probability that Kam Chancellor is on both teams?

b. What is the probability Kam Chancellor is placed on only one of these teams?

c. What is the probability that Kam Chancellor is not selected for the All-Pro team?

d. What is the probability that Kam Chancellor selected to at least one of the teams?

(3 pts) 13. Jacobs & Johnson, an accounting firm, employs 20 accountants, of whom 8 are CPAs. If a delegation of 4 accountants is randomly selected from the firm to attend a conference, what is the probability that 3 CPAs will be selected? (Round your answer to three decimal places.)

(4 pts) 14. Determine whether the events A and B are independent.

$P(A^c) = 0.2$, $P(B^c) = 0.3$, $P(A \cap B) = 0.56$

(4 pts) 15. The Public Housing Authority in Lubbock conducted a survey of 1000 families to determine the distribution of families by size. The results are given below.

Family Size	2	3	4	5	6	7	8
Frequency of Occurrence	308	205	200	82	54	20	131

a. Find the probability distribution of the random variable X, where X denotes the number of people in a randomly chosen family. (Enter your answers to three decimal places.)

b. Find the probability that a family chosen at random from those surveyed has less than five members. (Enter your answer to three decimal places.)

(4 pts) 16. From a panel of 100 graduate students of the Mathematics & Statistics Department at Texas Tech University, 35 are studying Applied Mathematics, 30 are studying Pure Mathematics and 35 are studying Statistics. Of the Statistics students, 40% are US citizens, whereas 50% of Applied Mathematics students are US citizens and 60% of Pure Mathematics students are US citizens.

a. Draw a tree diagram illustrating this problem.

b. If a graduate student from this panel is selected at random, what is the probability that he/she is a US citizen?

c. If a student selected at random is a US citizen, what is the probability he/she is studying Statistics?

(11 pts) 17. A flag company has a monthly fixed cost of \$9,008 and a variable cost of \$11 per flag produced. Each flag sells for \$27 each.

a. What is the cost function?

b. What is the revenue function?

c. What is the profit function?

d. What is the break-even point?

Finance Formulas

$A = P(1 + rt)$ Simple Interest

$A = P(1 + i)^n$ Compound Interest

$A = Pe^{rt}$ Compound Continuously

$r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$ Effective Rate, Compound

$r_{eff} = e^r - 1,$ Effective Rate, Continuous

$S = R\left[\frac{(1+i)^n - 1}{i}\right]$ Future Value

$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$ Present Value

$$i = \frac{r}{m}$$

$$n = mt$$

1 Final Key

1.

$$\begin{aligned} d(x) &= s(x) \\ -x^2+2x+233 &= 2x^2+2x+206 \\ 0 &= 3x^2-27 \\ 3x^2 &= 27 \\ x^2 &= 9 \\ x &= 3, -3(reject) \ \ (3pts) \end{aligned}$$

$$\begin{aligned} d(3) &= -(3)^2+2(3)+233 \\ &= 230 \ \ (2pts) \end{aligned}$$

$$(3, 230) \ \ (1pt)$$

2.a

$$\begin{aligned} \frac{-b}{2a} &= \frac{-(-400)}{2(4)} \ \ (2pts) \\ &= 50 \ units \ \ (1pt) \end{aligned}$$

2.b

$$\begin{aligned} C(50) &= 4(50)^2 - 400(50) + 15000 \ \ (2pts) \\ &= \$5000 \ \ (1pt) \end{aligned}$$

3.a

$$\begin{aligned} A(t) &= A_0e^{kt} \\ 100 &= 200e^{k27} \\ \frac{1}{2} &= e^{27k} \\ ln(\frac{1}{2}) &= ln(e^{27k}) \\ 27k &= ln(\frac{1}{2}) \ \ (3pts) \\ k &= \frac{ln(\frac{1}{2})}{27} \\ &= -0.02567 \ \ (1pt) \end{aligned}$$

3.b

$$A(t) = 200e^{\frac{ln(\frac{1}{2})}{27}*126} \quad (1pt)$$
$$= 7.87g \quad (1pt)$$
$$OR$$
$$A(t) = 200e^{-0.02567*126}$$
$$= 7.88g \quad (1pt)$$

4.

$$i = \frac{r}{m}$$
$$= \frac{.10}{4}$$
$$= .025$$
$$n = 4(3)$$
$$= 12$$
$$S = R[\frac{(1+i)^n - 1}{i}]$$
$$20,000 = R[\frac{(1+.025)^{12} - 1}{.025}]$$
$$R = \frac{20000(.025)}{(1+.025)^{12} - 1} \quad (4pts)$$
$$= \$1449.74 \quad (1pt)$$

5.a

$$16000(.25) = 4000$$
$$i = \frac{r}{m}$$
$$= \frac{.10}{12}$$
$$n = 36$$
$$P = R[\frac{1 - (1+i)^{-n}}{i}]$$
$$12000 = R[\frac{1 - (1+\frac{.10}{12})^{-36}}{(\frac{.10}{12})}]$$
$$R = \frac{12000(\frac{.10}{12})}{(1 - (1+\frac{.10}{12})^{-36})} \quad (3pts)$$
$$= \$387.21 \quad (1pt)$$

5.b

$$\$387.21(36) = \$13939.56 \quad (1pt)$$
$$\$13939.56 - \$12000.00 = \$1939.56 \quad (1pt)$$

6.

$$\begin{aligned}
A &= Pe^{rt} \\
2P &= Pe^{.055t} \\
2 &= e^{.055t} \\
ln(2) &= ln(e^{.055t}) \\
.055t &= ln(2) \\
t &= \frac{ln(2)}{.055} \quad (4pts) \\
t &= 12.6 \quad years \quad (1pt)
\end{aligned}$$

7.a

$$\begin{aligned}
i &= \frac{r}{m} \\
&= \frac{.07}{12} \\
n &= 12(20) \\
&= 240 \\
P &= R[\frac{1-(1+i)^{-n}}{i}] \\
&= 2000[\frac{1-(1+\frac{.07}{12})^{-240}}{(\frac{.07}{12})}] \quad (3pts) \\
&= \$257965.01 \quad (1pt)
\end{aligned}$$

7.b

$$\begin{aligned}
i &= \frac{.07}{12} \\
n &= 12(40) \\
&= 480
\end{aligned}$$

$$S = R[\frac{(1+i)^n - 1}{i}]$$

$$\begin{aligned}
257965.01 &= R[\frac{(1+\frac{.07}{12})^{480} - 1}{(\frac{.07}{12})}] \quad (3pts) \\
R &= \frac{257965.01(\frac{.07}{12})}{((1+\frac{.07}{12})^{480} - 1)} \\
&= \$98.28 \quad (1pt)
\end{aligned}$$

8.a

$$\begin{aligned}
245000 - 25000 &= 220000 \\
i &= \frac{r}{m} \\
&= \frac{.07}{12} \\
n &= mt \\
&= 12(15) \\
&= 180 \\
P &= R[\frac{1-(1+i)^{-n}}{i}] \\
220000 &= R[\frac{1-(1+\frac{.07}{12})^{-180}}{(\frac{.07}{12})}] \ \ (2pts) \\
R &= \frac{220000(\frac{.07}{12})}{(1-(1+\frac{.07}{12})^{-180})} \\
&= \$1977.42 \ \ (1pt)
\end{aligned}$$

8.b

$$\begin{aligned}
i &= \frac{.07}{12} \\
n &= 10(12) \\
&= 120 \\
P &= R[\frac{1-(1+i)^{-n}}{i}] \\
&= 1977.42[\frac{(1-(1+\frac{.07}{12})^{-120})}{(\frac{.07}{12})}] \ \ (2pts) \\
&= \$170,307.98 \ \ (1pt)
\end{aligned}$$

8.c

$$\begin{aligned}
&\$245000.00 - \$170307.98 \ \ (2pts) \\
&= \$74,692.02 \ \ (1pt)
\end{aligned}$$

8.d

$$\begin{aligned}
\$1977.42(180) &= \$355935.60 (2pts) \\
\$355935.60 - \$220000.00 &= \$135,935.60 \ \ (1pt)
\end{aligned}$$

9 (3pts)

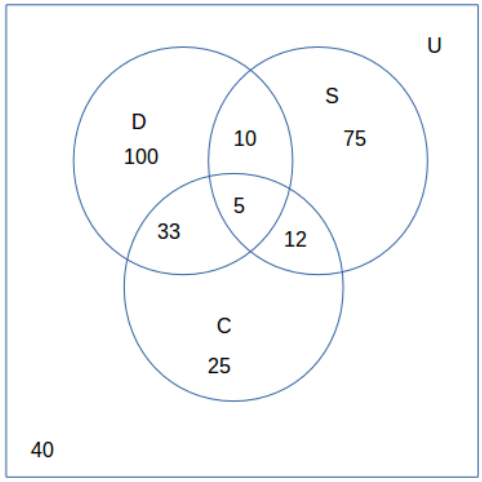


Figure 1: Figure for 9

9.a

$$40 \ \ (1pt)$$

9.b

$$100 + 75 + 25 = 200 \ \ (1pt)$$

9.c

$$12 \ \ (1pt)$$

10.

$$3 * 8 * 3 \ \ (3pts)$$
$$= 72 \ \ (1pt)$$

11.

$$C(7,2) * C(2,1) * C(3,1) * [C(3,1) + C(6,1)](5pts)$$
$$= 1134(1pt)$$

12.a

$$0.5 \ \ (1pt)$$

12.b

$$0.25 + 0.1 = 0.35 \ \ (1pt)$$

12.c

$$0.25 + 0.15 = 0.4 \ \ (1pt)$$

12.d

$$0.25 + 0.5 + 0.1 = 0.85 \ \ (1pt)$$

13.

$$\begin{aligned} P(3CPAs) &= \frac{P(8,3)C(12,1)}{P(20,4)} \ \ (2pts) \\ &= 0.139 \ \ (1pt) \end{aligned}$$

14.

$$\begin{aligned} P(A) &= 1 - P(A^c) \\ &= 1 - .2 \\ &= .8 \\ P(B) &= 1 - P(B^c) \\ &= 1 - .3 \\ &= .7 \\ P(A) * P(B) = P(A \cap B) & \\ .8(.7) &= 0.56 \\ .56 &= .56 \ \ correct \ \ (3pts) \end{aligned}$$

yes, they are independent $(1pt)$

15.a

X	2	3	4	5	6	7	8
P(X=x)	.308	.205	.200	.082	.054	.020	.131

15.b

$$\begin{aligned} P(X < 5) &= P(X = 2) + P(X = 3) + P(X = 4) \\ &= .308 + .205 + .200 \ \ (1pt) \\ &= 0.713 \ \ (1pt) \end{aligned}$$

16.a (2pts)

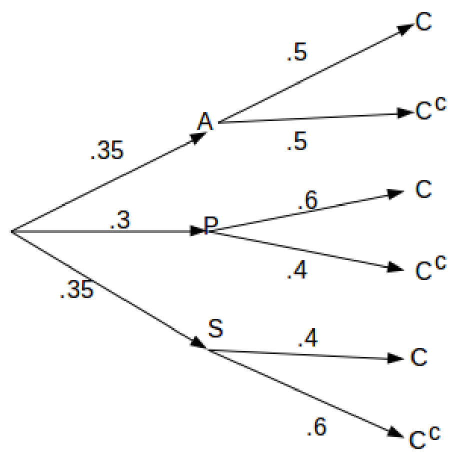


Figure 2: Figure for 16.a

16.b

$$
\begin{aligned}
P(C) &= .35(.5) + .3(.6) + .35(.4) \\
&= .495 \ \ (1pt)
\end{aligned}
$$

16.c

$$
\begin{aligned}
P(S|C) &= \frac{P(S \cap C)}{P(C)} \\
&= \frac{.35(.4)}{.495} \\
&= 0.283 \ \ (1pt)
\end{aligned}
$$

17.a

$$
\begin{aligned}
C(X) &= V(X) + F(X) \\
&= 11x + 9008 \ \ (2pts)
\end{aligned}
$$

17.b

$$
\begin{aligned}
R(X) &= sx \\
&= 27x \ \ (2pts)
\end{aligned}
$$

17.c

$$\begin{aligned} P(X) &= R(X) - C(X) \\ &= 27x - (11x + 9008) \\ &= 16x - 9008 \ \ (3pts) \end{aligned}$$

17.d

$$\begin{aligned} R(X) &= C(X) \\ 27x &= 11x + 9008 \\ 16x &= 9008 \\ x &= 563 \\ R(563) &= 27(563) \\ &= 15201 \ \ (3pts) \end{aligned}$$

$(563 flags, \$15201) \ \ (1pt)$