MATH 1330 Final Exam Fall 2015

Work each problem completely. Show your work and mark your final answer to receive full credit. All problems are weighted equally.

1. Barbara knows that she will need to buy a new car in 2 years. The car will cost \$15,000 by then. How much money should she invest now at 5%, compounded quarterly, so that she will have enough money to buy a new car?

2. Starting at the age of 50, Morgan puts \$1,600 at the end of each quarter into a retirement account that pays 7% interest compounded quarterly. When she reaches age 60, she withdraws the entire amount and places it in a mutual fund account that pays 9% compounded monthly. She does not make anymore quarterly contribution. How much is in the account when she reaches age 68?

3. At age 21, you now have access to your trust fund of \$250,000. The money is in an account that pays 6.1% interest compounded semiannually. You want to make equal semiannual withdrawals from the trust fund so that the money (principal and interest) lasts exactly 25 years. Determine the amount of each semiannual withdrawal.

4. Determine the interest rate needed for an investment of \$10,000 to grow to an amount of \$18,000 in 5 years if the interest is compounded monthly.

5. A manufacturer of computer chips has a monthly fixed cost of \$20,000, a production cost of \$30 per unit, and a selling price of \$50 per unit.

(a) What is the cost function $C(x)$ for producing x units?

(b) What is the revenue function $R(x)$ for selling x units?

(c) What is the profit function $P(x)$ for producing and selling x units?

(d) Find the break-even revenue.

6. A building contractor buys 70% of his cement from supplier A and 30% from supplier B. A total of 90% of the bags from A arrive undamaged, while a 95% of the bags from B arrive undamaged.

(a) Find the probability that a damaged bag is from supplier A.

(b) Are the events that the cement is from A and the cement is damaged independent? Justify your answer by showing your work.

7. A survey of 149 families showed that 59 had a dog; 46 had a cat; 19 had a dog and a cat; 63 had neither a cat nor a dog. How many families surveyed had a dog only? Use a Venn diagram to answer the question.

8. Suppose 12% of the population are 61 or over, 29% of those 61 or over have loans, and 52% of those under 61 have loans. Find the probabilities that a person fits into the following categories.

(a) 61 or over and has a loan

(b) Has a loan.

9. A factory wants to ship 5 machines to its client. However there are 3 defective machines in a lot of 15.

(a) Find the probability that the shipment contains exactly 1 defective machine.

(b) Find the probability that there is at least one defective machine in the shipment.

10. Suppose a local symphony decides to raise money by raffling an HD television worth \$400, a dinner for two worth \$80, and 2 CDs worth \$20 each. A total of 2000 tickets are sold at \$1 each. Find the expected payback for a person who buys one ticket in the raffle.

11. A poker company assembles three different poker sets. Each Royal Flush poker set contains 1000 poker chips, 10 decks of cards, 4 dice, and 2 dealer buttons. Each Deluxe Diamond poker set contains 600 chips, 5 decks of cards, 2 dice, and one dealer button. The Full House poker set contains 300 poker chips, 5 decks of cards, 2 dice, and one dealer button. The company has 2,900,000 poker chips, 25,000 decks of cards, 10,000 dice, and 7,000 dealer buttons in stock. They earn a profit of \$38 for each Royal Flush poker set, \$22 for each Deluxe Diamond set, and \$12 for each Full House poker set. Find the maximum profit and the number of each type of poker set to maximize the profit by answering the following questions.

(a) Complete the table with all appropriate data filled in.

	Royal Flush x_1	Deluxe Diamond x_2	Full House x_3	Totals
Poker Chips		600		
	10			$\leq 25,000$
Dice		2		
	2			$\leq 7,000$
Profit		22		z

(b) Add the slack variables appropriate to each constraint.

(c) Rearrange the objective function in preparation for forming the initial simplex tableau.

(d) Form the initial simplex tableau. Make sure all appropriate labels are clearly written. Do not solve the matrix at this point.

(e) The following is the simplex tableau after one pivot has been performed. If necessary, continue to pivot until you have reached the final simplex tableau that will produce the optimal solution. Clearly show this final matrix.

$$\left[\begin{array}{cccccccc|c} 0 & 100 & -200 & 1 & -100 & 0 & 0 & 0 & 400,000 \\ 10 & 5 & 5 & 0 & 1 & 0 & 0 & 0 & 25,000 \\ 0 & 0 & 0 & 0 & -2 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 5 & 0 & 10,000 \\ \hline 0 & -15 & 35 & 0 & 19 & 0 & 0 & 5 & 475,000 \end{array}\right]$$

(f) List the basic variables and their values.

(g) List the non-basic variables and their values.

(h) To produce a maximum profit, how many Royal Flush sets, how many Deluxe Diamond sets, and how many Full House sets should be produced? What is the maximum profit? Once this maximum profit is achieved, how many poker chips are left in stock, how many decks of cards are left in stock, how many dice are left in stock, and how many dealer buttons are left in stock?

12. A dietician is planning a snack package of fruit and nuts. Each ounce of fruit will supply zero units of protein, 2 units of carbohydrates, and 1 unit of fat, and will contain 21 calories. Each ounce of nuts will supply 3 units of protein, 1 unit of carbohydrates, and 2 units of fat, and will contain 28 calories. Every package must provide at least 12 units of protein, at least 8 units of carbohydrates, and no more than 11 units of fat. By answering the following questions, find the number of ounces of fruit and number of ounces of nuts that will meet the requirement with the least number of calories. What is the least number of calories possible in a package?

(a) Complete the table with all appropriate data filled in. Be sure to identify your variables in the table.

	Fruit x_1	Nuts x_2	Totals
Protein	0		≥ 12
		1	
Fat	1		≤ 11
	21		z

(b) Write out the objective function that is to be minimized.

(c) Write out the constraints.

(d) Graph the feasible region. Be sure to shade the region clearly.

(e) Is the feasible region bounded or unbounded?

(f) Complete the corner point chart.

Corner Point	Calories
$\left(\frac{5}{3}, \frac{14}{3}\right)$	165.7

(g) How many ounces of each (fruit and nuts) should the dietician put in each package? What is the least number of calories possible in a package?