1330 Final Exam, Fall 2016

You must show your work, and the work you show must yield the answer you obtain, if you are to receive credit. Present the problems in your blue book in the order that they occur on the exam: problem 1 first, problem 2 second, etc. Allow at least one full page for each problem. Note that there is a 20 point bonus question (#11).

Math of Finance Formulae

Simple Interest: $FV = PV(1 + rt)$

Compound Interest: $FV = PV(1 + i)^n, \ FV = PVe^{rt}$

Effective Rate: $r_E = \left(1 + \dfrac{r}{m}\right)^m - 1, r_E = e^r - 1$,

Annuities: $FV = PMT\dfrac{(1+i)^n - 1}{i}$

$$PV = PMT\dfrac{1 - (1+i)^{-n}}{i}$$

t = number of years
r = annual percentage rate
m= number periods per year
i=interest rate per period
n = total number of periods
r_E= effective rate
FV = future value
PV = present value
PMT = periodic payment

1. (20p) **Revenue.** The owner of an apartment building can rent all 60 apartments if she charges $650 per month, but she rents one fewer apartment for each $25 increase in monthly rent.
   a. (4p) Construct a table that gives the revenue generated if she charges $650, $675, and $700.
   b. (4p) Write an equation, $R(x)$, that gives the revenue from rental of the apartments if she makes x increases of $25 in the rent.
   c. (4p) Provide a sketch of the graph $y = R(x)$ and label the x and y intercepts. Use your calculator with window $-26 \le x \le 60$ and $0 \le y \le 50.000$.
   d. (4p) Find the rent she must charge to maximize her revenue.
   e. (4p) What is the maximum revenue she can get?

2. (20p) **Cost-Revenue-Profit**. It costs a company a fixed cost of $27,000 to set up a production line. Once the line is operational the production cost is $20 per unit. The company can sale its commodity for $50 per unit. Let x represent the number of units.
   a. (4p) What is the total cost function?
   b. (4p) What is the total revenue function?
   c. (4p) What is the profit function?
   d. (4p) What is the marginal profit? Explain
   e. (4p) What is the company's break even quantity?

3. (20) **Supply and Demand.** Suppose the demand for a particular product is given by the equation $p = -2q + 20$ and the supply equation is given by $p = 3q + 5$, where p is the price per unit and q is the number of units.
   a. (4p) Sketch the graphs of the demand and supply equations on the same coordinate axis with p being on the vertical axis with $0 \le p \le 30$ and q on the horizontal axis with $0 \le q \le 10$.
   b. (4p) Explain why we expect the demand equation to have negative slope and the supply equation to have positive slope.
   c. (4p) Is there a shortfall (more demand than supply) or a surplus (more supply than demand) at the unit price of 10$? Explain.
   d. (4p) Find the point of equilibrium (where supply and demand are equal).
   e. (4p) Suppose a 0.25$ tax is placed on each unit. What will be the new supply equation?

4. (20p) **Cost to Reduce Pollution.** Suppose the cost $C(p)$ (**in hundreds of dollars**) of removing p percent of the particulate pollution from the smokestacks of an industrial plant is given by

$$C(p) = \frac{7300p}{100 - p}.$$

a. (4p) For what value(s) of p is the function $C(p)$ not defined?
b. (4p) What is the domain of $C(p)$ in the context of this application?
c. (4p) Provide a sketch of the graph of the function $C(p)$ over the domain given in part b. ((In your calculator choose the y-min=0 and y-max=300,000.)
d. (4p) How much will it cost to remove 75% of the particulate pollution from the smokestacks?
e. (4p) Can the plant afford to remove all the particulate pollution from its smokestacks? Explain.

5. (2p) **Disposable Income.** The following table gives the total U.S. disposable income (in billions of 2005 dollars) for selected years from 2010 and projected to 2040.

Year	2010	2015	2020	2025	2030	2035	2040
Income (billions)	$10,017	$11,120	$12,655	$14,259	$15,948	$17,752	$19,785

a. (10p) Use x as the number of years past 2010 and write the equation of the function that is the best linear fit for these data.
b. (5p) What does the model predict for the disposable income in 2018?
c. (5p) Interpret the slope of the model found in part (a).

6. (20p) **Open Economy.** An economy has an agricultural industry and a textile industry. Each unit of agricultural output requires 0.4 unit of agricultural input and 0.1 units of textiles input. Each unit of textiles output required 0.1 unit of agricultural input and 0.2 unit of textiles input.
a. (4p) Write the technology matrix for this economy.
b. (4p) How many units of textile products are required to produce 100 units of agricultural products?
c. (4p) How many units of agricultural products and textile products are required to product 100 units textile products?
d. (4p) If surpluses of 5 units of agriculture products and 100 units of textiles are desired, what is the technology equation?
e. (4p) Find the gross production of each industry.

7. (20p) **Linear Programing.** Nwjet, Inc. manufactures inkjet printers and laser printers. The company has the capacity to make 70 printers per day, and it has 120 hours of labor per day available. It takes 1 hour to make an inkjet printer and 3 hours to make a laser printer. The profits are $40 per inkjet printer and $60 per laser printer.
   a. (4p) Let x represent the number of inkjet printers made and let y represent the number of laser printers made. What is the profit function? (Also called the "objective function" in this context.)
   b. (4p) What are the constraint inequalities related to the companies' manufacturing capacity and labor restriction?
   c. (4p) Sketch the feasible region.
   d. (4p) What of each type of printer should be made to give maximum profit?
   e. (4p) What is the maximum profit.

8. (20p) **Simple Interest.** Bill bought a $10,000, 5 year certificate of deposit (CD) that would earn 3% annual simple interest. Two years before the CD was due to mature, Bill needed his CD money, so a friend agreed to lend him money and receive the value of the CD when it matured.
   a. (5p) How much interest will the CD generate upon maturity?
   b. (5p) What will be the value of the CD (its future value) when it matures?
   c. (5p) If the agreement allowed the friend to earn an annual interest rate of 5% return on his loan to Bill, how much did Bill receive from his friend?
   d. (5p) How much would Bill have gotten if he had been able to simply cash in his CD in just 3 years, rather than having to wait 5 years to cash it in?

9. (20p) **Compound Interest**. Suppose $5000 is invested in a start-up enterprise with estimated return at an annual rate of 6% .
   a. (4p) How much will the investment be worth after 5 years if the interest is compounded quarterly?
   b. (4p) What is the Annual Percentage Yield in case **a**?
   c. (4p) How much will the investment be worth after 5 years if the interest is compounded continuously?
   d. (4p) What is the Annual Percentage Yield in case **c**?
   e. (4p) How long will it take for the investment to double if the interest is compounded continuously?

10. (20p) **Future and Present Value of an Annuity.** A small business owner contributes $1000 at the end of each month into an account that earns an annual rate of 5% compounded monthly.
    a. (5p) How much is in the account (its future value) after 5 years?
    b. (5p) How much would the business owner need to contribute each month if she wanted the value in 5 years to be $100,000?
    c. (5p) Suppose she currently has an account valued at $200,000 (its current value) which is also growing an annual rate of 5% compounded monthly. How much can she withdraw monthly (in equal amounts) from this current account if she wants to deplete the account at the end of 5 years.
    d. (5p) How much would see need to add to this second account if she wanted to withdraw $2000 each month for 5 years?

11. (20p bonus) **Amortization.** A woman buys house for $200,000. She makes a $50,000 down payment and amortizes the rest of the purchase price with monthly payments over the next 30 years with an annual interest rate of 4% compounded monthly.
    a. (4p) What is the size of each monthly payment?
    b. (4p) How much of the first payment is interest? (Remember that the first payment includes the interest due from holding $200,000 for one month.)
    c. (4p) How much does she still owe after making the first payment?
    d. (4p) What is the total amount of money she will have paid over the life of the loan?
    e. (4p) What is the total interest she will have paid over the life of the loan?