

### 1330 Final Exam, Spring 2017

You must show your work, and the work you show must yield the answer you obtain, if you are to receive credit. Present the problems in your blue book in the order that they occur on the exam: problem 1 first, problem 2 second, etc. Allow at least one full page for each problem. Note that there is a 20 point bonus question (#11).

#### Math of Finance Formulae

Simple Interest:  $S = P(1 + rt)$

Compound Interest:  $S = P(1 + i)^n$ ,  $S = Pe^{rt}$

Effective Rate:  $AYP = \left(1 + \frac{r}{m}\right)^m - 1$ ,  $AYP = e^r - 1$

Annuities: Future value  $S = R \left[ \frac{(1+i)^n - 1}{i} \right]$

Present value  $A = R \left[ \frac{1 - (1+i)^{-n}}{i} \right]$

t = number of years
r = annual percentage rate
m = number periods per year
i = interest rate per period
n = total number of periods
AYP = effective rate
S = future value
A = present value
R = periodic payment

- (20p) **Cost-Revenue-Profit.** A manufacturer of DVD players has monthly fixed costs of \$9800 and variable costs of \$60 per unit for one particular model. The company sells this model to dealers for \$100 each.
  - (4p) What is the total cost function?
  - (4p) What is the total revenue function?
  - (4p) What is the profit function?
  - (4p) What is the marginal profit? Explain what *marginal profit* means.
  - (4p) What is the company's break even quantity?
- (20p) **Supply and Demand.** The supply and demand functions for a certain brand of shoes are given by the equations  $2p + 5q = 200$  and  $p - 2q = 10$ .
  - (5p) Sketch the graphs of these two functions on the same coordinate axis with  $p$  being on the vertical axis with  $0 \leq p \leq 110$  and  $q$  on the horizontal axis with  $0 \leq q \leq 40$ .
  - (5p) Label the graphs D for demand and S for supply. Explain how you know which is supply and which the demand.
  - (5p) Compute the equilibrium point (where supply equals demand). Label it on the graphs.
  - (5p) Is there a surplus or shortfall at the price of \$60? Explain.
- (20p) **Revenue.** An agency charges \$150 per person for a trip to a concert if 30 people travel in a group. But for each person above the 30, the amount charged each traveler will be reduced by \$2.00.
  - (4p) Construct a table giving the price per traveler and the total revenue for 30, 31, 32, and 33 travelers.
  - (4p) Write the equation that gives the revenue  $R(x)$  for  $30 + x$  travelers.
  - (4p) Sketch the graph  $y = R(x)$  and label the  $x$  and  $y$  intercepts. Use your calculator using the settings  $-15 \leq x \leq 30$  and  $-10 \leq y \leq 800$ .
  - (4p) Find the value of  $x$  that yields the maximum revenue.
  - (4p) Find the maximum revenue.
- (20p) **Population Data.** The following table gives the projected population, in thousands, of U.S. females under the age of 18.

Year	Population (1000s)	Year	Population (1000s)
2020	37,222	2045	41,067
2025	38,201	2050	41,963
2030	39,244	2055	42,855
2035	39,809	2060	43,610
2040	40,352		

Source: U.S. Census Bureau

- (5p) Use your calculator to create a linear function that models the projected population,  $y$ , in thousands, as a function of the number of years past 2010 ( $x=10, x=15, x=20, x=25$  etc.). Round to 3 decimal places.
  - (5p) Find and interpret the slope of the model.
  - (5p) What is the projected population in the year 2065?
  - (5p) When will the population be 50,000,000 according to this model?
5. (20p) **Manufacturing.** A manufacturer of table saws has three models. Deluxe, Premium, and Ultimate, which must be painted, assembled, and packaged for shipping. The table gives the number of hours required for each of these operations for each type of table saw.

	Deluxe	Premium	Ultimate
Painting	1.6	2	2.4
Assembly	2	3	4
Packaging	0.5	0.5	1

- (10p) Let  $x$  represent the number of Deluxe table saws,  $y$  the number of Premium, and  $z$  the number of Ultimate. Suppose the manufacturer has 96 hours available per day for painting, 156 hours for assembly, and 37 hours for packaging. Find the set of equations (three equations in the three variables  $x, y, z$ ) that represents this scenario.
  - (5p) Write this system in matrix form:  $AX = B$ .
  - (5p) How many of each model can be produced each day?
6. (20p) **A Closed Economy.** A closed model for an economy has a manufacturing industry, utilities industry, and households industry. Each unit of manufacturing output uses 0.5 unit of manufacturing input, 0.4 unit of utilities input, and 0.1 unit of households input. Each unit of utilities output requires 0.4 unit of manufacturing input, 0.5 unit of utilities input, and 0.1 of households input. Each unit of households output requires 0.3 unit each of manufacturing and utilities input and 0.4 unit of households input.
- (5p) Write the technology matrix for this closed model of the economy.
  - (5p) Write the technological equation  $(I - A)X = O$  for this model.
  - (5p) Find the gross production for each industry.
  - (5p) Explain the results in part c.
7. (20p) **Simple Interest.** An investor owns several apartment buildings. The taxes on these buildings total \$30,000 per year and are due before April 1. The late fee is  $\frac{1}{2}\%$  per month up to 6 months, at which time

the buildings are seized by the authorities and sold for back taxes. Suppose the investor has \$30,000 available on March 31.

- a. (5p) If the investor were to invest the \$30,000 at 8%, for the six months, what is the total amount of money she will receive?
  - b. (5p) What is the amount of interest she will have earned?
  - c. (5p) What is the total amount of the late fee if she pays the taxes on September 30?
  - d. (5p) Should she pay the taxes on March 31 or invest the money and pay the taxes and fee on September 30? Explain.
8. (20p) **Compound Interest.** Grandparents want to make a gift of \$100,000 for their grandson's 20<sup>th</sup> birthday.
- a. (5p) How much would they have to invest on the day of their grandson's birthday if their investment earned 10% compounded continuously?
  - b. (5p) What is the effective rate for this investment?
  - c. (5p) If they had \$20,000 to invest, what annual rate would they need to reach the same goal?
  - d. (5p) How much would they have to invest at the rate in art (c) if they waited until the grandson's 6<sup>th</sup> birthday?
9. (20p) **Retirement Goals.** Joe is 30 years old and expects to retire at age 70, and wants to be able to withdraw \$10,000 per month from his retirement account.
- a. (10p) He anticipates being able to invest his total retirement account in a company that returns an annual interest of 7% compounded monthly. How much principle will he need to invest at age 70 if he expects to live to 90? (What present value at age 70 does he need in order to withdraw \$10,000 per month for 20 years?)
  - b. (10p) Joe has opened a 401K retirement account that guarantees a 5% annual return. How much does Joe have to put into this account each month in order to meet the his goal on his 70<sup>th</sup> birthday?
10. (20p) **Home Mortgage.** A man buys a house for \$350,000. He makes a down payment of \$150,000 and amortizes the rest of the purchase price with monthly payments for 30 years. His interest rate is 4.5% compounded monthly.
- a. (5p) What are his monthly payments?
  - b. (5p) How much does he owe after the first payments?
  - c. (5p) What is the total amount he will end up paying for the house, including his down payment?
  - d. (5p) How much of this total is due to interest?
11. (20p) **Linear Programming.** Nwjet, Inc. manufactures inkjet printers and laser printers. The company has the capacity to make 70 printers per day, and it has 120 hours of labor per day available. It takes 1 hour to make an inkjet printer and 3 hours to make a laser printer. The profits are \$40 per inkjet printer and \$60 per laser printer.
- a. (4p) Let  $x$  represent the number of inkjet printers made and let  $y$  represent the number of laser printers made. What is the profit function? (Also called the "objective function" in this context.)
  - b. (4p) What are the constraint inequalities related to the company's manufacturing capacity and labor restriction?
  - c. (4p) Sketch the feasible region.
  - d. (4p) How many of each type of printer should be made to give maximum profit?
  - e. (4p) What is the maximum profit?