Math of Finance Formulas

Simple Interest: \( S = P(1 + rt) \)
Compound Interest: \( S = P(1 + i)^n \), \( S = Pe^{rt} \)
Effective Rate: \( APY = \left(1 + \frac{r}{m}\right)^m - 1 \), \( APY = e^r - 1 \)

Annuities: Future value \( S = R\left(\frac{(1 + i)^n - 1}{i}\right) \), Present value \( A = R\left(\frac{1 - (1 + i)^{-n}}{i}\right) \)

\( t \) = number of years, \( r \) = annual percentage rate, \( m \) = number of periods per year, \( i \) = interest rate per period, \( n \) = total number of periods, \( R \) = periodic payment.

1. (10p) Use the given feasible region determined by the constraint inequalities to find the maximum and minimum of the given objective function \( C = 10x + 2y \) (if they exist). (If an answer does not exist, enter DNE.)

(a) (5p) Maximum C
(b) (5p) Minimum C

2. (10p) To buy a Treasury bill (T-bill) that matures to \$7,000 in 9 months, you must pay \$6,520. What annual simple interest rate does this earn? (Round your answer to one decimal place in percent.)
3. (10p) Suppose that an economy has three industries, fishing, agriculture, and mining, and the following matrix \( A \) is the technology matrix for this economy.

\[
A = \begin{bmatrix}
0.5 & 0.1 & 0.1 \\
0.3 & 0.5 & 0.2 \\
0.1 & 0.3 & 0.4 \\
\end{bmatrix}
\]

If surpluses of 110 units of fishing output and 50 units each of agricultural and mining goods are desired, find the gross production of each industry.

4. (20p) A person invests $3,000 at the end of each month for 10 years in an account that earns 6%, compounded monthly. After the initial 10 years, no additional contributions are made, but the investment continues to earn 6%, compounded monthly, for an additional \( 18 \frac{1}{2} \) years.

(a) At the end of the first 10 years how much does the person have?

(b) How much does the person have at the end?

5. (10p) For the total cost function \( C(x) = 3600 + 100x + 2x^2 \) and the total revenue function \( R(x) = 500x - 2x^2 \), find the number of units that maximizes profit and find the maximum profit.

6. (20p) Suppose an individual makes an initial investment of $2500 in an account that earns 7.8%, compounded monthly, and makes additional contributions of $100 at the end of each month for a period of 12 years. After these 12 years, this individual wants to make withdrawals at the end of each month for the next 5 years (so that the account balance will be reduced to $0).

(a) (5p) How much is in the account after the last deposit is made?

(b) (5p) How much is deposited?

(c) (5p) What is the amount of each withdrawal?

(d) (5p) What is the total amount withdrawn?

7. (20p) Suppose you are selling birthday cards and you have a fixed cost of $170 and a variable cost of $5.50 per unit. The birthday cards are sold for $10 per card.

(a) (5p) What is the total cost function?

(b) (5p) What is the total revenue function?

(c) (5p) What is the profit function?

(d) (5p) What is the break-even quantity?
8. (20p) Clark and Lana take a 30-year home mortgage of $121,000 at 7.8%, compounded monthly. They make their regular monthly payments for 5 years, then decide to pay $1000 per month.

(a) (5p) Find their regular monthly payment. (Round your answer to the nearest cent.)

(b) (5p) Find the unpaid balance when they begin paying the $1000. (Round your answer to the nearest cent.)

(c) (5p) How many payments of $1000 will it take to pay off the loan?

(d) (5p) How much interest will they save by paying the loan using the number of payments from part (c)? (Round your answer to the nearest cent.)

9. (10p) Suppose the demand function for $q$ thousand units of a certain commodity is given by

$$p = 30(3^{-q/2}).$$

(a) (5p) At what price per unit will the demand equal 4000 units?

(b) (5p) How many units, to the nearest thousand units, will be demanded if the price is $17.32?

10. (10p) By using Social Security Administration data for selected years from 2012 and projected to 2050, the U.S. consumer price index (CPI) can be modeled by the function

$$C(t) = 92.7e^{0.0271t}$$

where $t$ is the number of years past 2010. With the reference year as 2012, a 2016 CPI = 108.58 means goods and services that cost $100.00 in 2012 are expected to cost $108.58 in 2016.

(a) (5p) Find $C(15)$ and explain its meaning.

(b) (5p) What is the CPI value in 2020?

11. (10p) The Goldsmiths purchased a house 10 years ago for $200,000. They made a down payment of 20% of the purchase price and secured a 30-year conventional home mortgage at 9% per year on the unpaid balance. The house is now worth $380,000.

(a) (5p) Determine their monthly payment.

(b) (5p) How much equity do they have in their house now (after making 120 monthly payments)?

12. (10p) How long does it take an investment of $10,000 to double if it is invested at

(a) (5p) 8% compounded annually?

(b) (5p) 8% compounded continuously?