

### Math 1331 Final Exam Fall 2010

Show all work! A correct answer with no work is worth 1 point. Please circle all your answer and work all questions in order. Answer all word problems with complete sentences. Use 4 decimal places for your answers. All parts are worth 5 points.

1. Acne toy company finds the price function to be given by

$$p(x) = \frac{2x^2 + x}{x^2 - 3x}$$

- a) Find the revenue function and simplify.  
b) The cost function is defined as

$$C(x) = \frac{4x^2 - 4x - 3}{x - 3}$$

Find the profit function and simplify.

2. The following data shows the relationship between a student's homework average and their final grade.

Homework Average	95.75	79.64	59.64	67.43	88.34
Final Average	99.64	82.34	49.32	70.56	92.47

- a) Find the quadratic function that best represents the data. State  $r^2$ .  
b) Find the maximum Homework Average and the Final Average. *Hint:* The Homework average can be over 100.
3. The concentration of a certain drug (in mg/cm<sup>2</sup>) in a patient's bloodstream  $t$  hour after the injection is given by the following function

$$C(t) = \frac{-2t^2 + t - 5}{t^3 + 4t^2 + 5t - 3}$$

How much of the drug remains in the system as time increases without bound? Explain your answer.

4. The profit (in millions) function for a company is given by

$$P(x) = \frac{x^2 - 100}{x^2 - 15x + 50}$$

where  $x$  is the number of units produced and sold (in hundreds). What happens to the company's profit  $P(x)$  as the number of units sold approaches 1000? That is, evaluate

$$\lim_{x \rightarrow 10} P(x)$$

5. Given the following revenue functions, find the marginal revenue.

a)  $R(x) = 7x^4 - 3x^2 + 5x^{-6} + x + 8$

b)  $R(x) = (x^7 + e^{4x}) \ln(3x^2 + 1)$

6. The number of people visiting Six Flags Over Texas is given by the function  $f(x)$ . Here  $f(x)$  is measured in thousands and  $x$  is the week number the park is open

$$f(x) = \sqrt[3]{(4x^2 + x - 9)^4}$$

- a) How fast is the number of visitors of Six Flags Over Texas is changing?

- b) How fast is the number of visitors changing in week 10? That is, calculate  $f'(10)$ .

7. The weekly cost for making Christmas tree lights is represented by the function

$$C(x) = x^4 - 2x^3 - x^2 + 7x + 62$$

where the cost is in hundreds of dollars and  $x$  is Christmas tree light sets. Find the cost of producing the 10th set of Christmas tree lights.

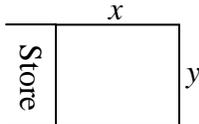
8. If exactly 200 people sign up for a charter flight, Leisure World Travel Agency charges \$320 per person. However, if more than 200 people sign up for the flight (assume this is the case), then each fare is reduced by \$2 for each additional person. How many passengers will result in a maximum revenue for the travel agency? What is the maximum revenue? *Hint:* Let  $x$  denote the number of passengers above 200.

9. The altitude (in feet) of a rocket  $t$  minutes into the flight is given by the following function

$$s = f(t) = -t^4 + t^3 + 200t + 7$$

At what time does the rocket reach its maximum velocity? What is the maximum velocity? When is the velocity increasing and decreasing? *Hint:* Find the inflection point.

10. A new restaurant wants to enclose a rectangular patio for customers to eat outdoors on nice days. The cost of the stone to make a fence is \$4 per foot. The area to be enclosed is  $90\text{ft}^2$ . What are the dimensions of the rectangular area to minimize the cost?



11. Evaluate the following integrals.

a)

$$\int \frac{x-3}{\sqrt{x^2-6x+5}} dx$$

b)

$$\int_1^2 \left( 3x^4 - 3e^x + \frac{1}{x^5} + \frac{5}{x} + 2 \right) dx$$

12. Carlota Music Company estimates that marginal cost of manufacturing its Christmas series CD is given by the following in dollars/week when the level of production is  $x$  CD/week.

$$C'(x) = 0.04x^2 - 3x + 201$$

The fixed costs incurred by Carlota are \$2500/week. Find the total weekly cost  $C(x)$  incurred by Carlota in manufacturing  $x$  CD/week.

13. Annual sales (in millions of units) of Christmas wrapping paper are expected to grow in accordance with the following function, where  $t$  is measured in years, with  $t = 0$  corresponding to 2000.

$$f(t) = 0.21t^3 + 0.12t + 4.26; \quad (0 \leq t \leq 6)$$

How many packages of Christmas wrapping paper will be sold over the 7 year period between the beginning of 2000 and the end of 2006?

14. Find the area between the two curves  $f(x) = 3x + 1$  and  $g(x) = x^2 - 3$ .

15. The demand function for a can of popcorn is given by the following equation where  $p$  is the unit price in dollars and  $x$  is the quantity demanded each week, measured in units of a thousand.

$$p = -2x^2 - 10x + 16$$

Determine the consumers' surplus if the market price is set at \$4/can.