MATH 1331 Final Exam, Fall 2011

Version A

1. The following table shows the net sales, in millions of dollars, for a manufacturer of laptop computers during the last five years.

Year, x	1	2	3	4	5
Net Sales, y	426	437	460	473	477

Use linear regression to find the least squares line for this data.

a) $y = 426x + 12.75$

b) $y = 12.75x + 426$

c) $y = 413.2x + 13.8$

d) $y = 13.8x + 413.2$

e) $y = 3.0x + 454.6$

2. An office supply company estimates that the total cost of making x ergonomic desk chairs, in dollars per year, is given by

$$C(x) = 10.3\sqrt{x} + 52.5x$$

What happens to the **AVERAGE** cost per chair when x becomes very large (i.e., what is the limit of the average cost as x goes to infinity)?

a) it approaches 52.5

b) it approaches 57.65

c) it becomes infinitely large

d) it approaches 0

e) it approaches 62.8

3. The position s (in feet) of a prototype SUV t seconds after starting a particular test is given by the following function:

$$s(t) = -2t^3 + 13t^2 - 10t$$

What is the SUV's velocity (i.e., at what rate is its position changing) after 4 seconds?

a) 36 ft/s

b) -2 ft/s

c) 70 ft/s

d) 8 ft/s

e) -22 ft/s

4. Scientists in a small developing nation have determined that the number of births per capita, B, in the nation is given by

$$B(x) = \sqrt{60 - 8x - 0.1x^2} \qquad (0 \leq x < 7)$$

where x is the current population in millions. Find the rate at which $B(x)$ is changing (rounded to two decimal places) when the population is 6 million people.

a) -1.59 births/capita per million people

b) 2.90 births/capita per million people

c) 0.17 births/capita per million people

d) -4.80 births/capita per million people

e) -2.90 births/capita per million people

5. A publishing company finds that the demand for one of their calculus study guides is modeled by the function

$$d(x) = \frac{35x}{2 + 0.2x^2}$$

where $d(x)$ is the quantity demanded per month (measured in units of a thousand) and x is the unit price (in dollars). At what rate is the demand for study guides changing when the price per book is \$12?

a) -30,455 books/month per \$1 increase

b) 7,291 books/month per \$1 increase

c) -989 books/month per \$1 increase

d) -938,000 books/month per \$1 increase

e) 13,636 books/month per \$1 increase

6. Find the slope of the tangent line to the graph of

$$f(x) = x^2 e^{-3x}$$

for all x.

a) $xe^{-3x}(2 + x)$

b) $2xe^{-3x}$

c) $x^2e^{-3} + 2xe^{-3x}$

d) $-6xe^{-3x}$

e) $xe^{-3x}(2 - 3x)$

7. The price (in dollars) charged by a large retailer for package of mechanical pencils is given by

$$p = -.3x + 9.4$$

where x is the demand for pencils in millions of packages. How many packages does the retailer need to sell to maximize their **revenue**?

a) about 31,333,333

b) about 15,666,667

c) about 16

d) about 31

e) as many as possible

8. Find the rate at which the graph of the following function is changing:

$$g(x) = \ln\left(2x^3 - 4x^2 + 6\right)$$

a) $\dfrac{1}{2x^3 - 4x^2 + 6}$

b) $\dfrac{6x^2 - 8x}{\ln\left(2x^3 - 4x^2 + 6\right)}$

c) $\dfrac{2x^3 - 4x^2 + 6}{6x^2 - 8x}$

d) $\dfrac{3x^2 - 4x}{x^3 - 2x^2 + 3}$

e) $\ln\left(6x^2 - 8x\right)$

9. What is the absolute minimum value obtained by

$$f(x) = -\frac{2}{3}x^3 - \frac{3}{2}x^2 + 9x + 7$$

on the interval [-5,4].

a) -3.0

b) -23.7

c) -29.1

d) -15.5

e) no minimum

10. A bed and breakfast has determined that their annual revenue is related to the money they spend on advertising by the equation

$$R(x) = -0.003x^3 + 1.35x^2 + 2x + 8000$$

where the revenue R and the amount spent on advertising x are both in thousands of dollars. Find the amount spent on advertising at the company's point of diminishing returns (to the nearest thousand dollars).

a) \$301,000
b) \$205,000
c) \$29,000
d) \$150,000
e) \$8,000

11. Find the marginal profit function for a manufacturer to make x graphing calculators if the profit function is given by

$$P(x) = \frac{7}{\sqrt{x}} + 13x$$

a) $-\dfrac{7}{x^{-1/2}} + 13$

b) $\dfrac{7x}{x^{3/2}} + \dfrac{13}{2}x^2$

c) $7x^{3/2} + \dfrac{13}{2}x^2$

d) $-\dfrac{7}{2}x^{1/2} + 13$

e) $-\dfrac{7}{2x^{3/2}} + 13$

12. Sir Isaac's Wonderland is a small amusement park that caters exclusively to private parties and special occasions. The park charges \$64.50 per person for an all day pass for parties of up to 15 people. However, if there are more than 15 people in a party, then each person's ticket price is reduced by \$1.50 for each additional person. (For example, with a group of 17 people, the tickets would cost \$61.50 per person.) Determine how many people in a party will result in the maximum revenue for the amusement park.

a) as many as possible
b) 14 people
c) 43 people
d) 37 people
e) 29 people

13. Find the indefinite integral

$$\int \frac{x^3 - x}{x^4 - 2x^2}\,dx$$

a) $\dfrac{\ln|x^4 - 2x^2|}{4} + C$

b) $\dfrac{(x^4/4) - (x^2/2)}{(x^5/5) - (2x^3/3)} + C$

c) $4\ln|x^4 - 2x^2| + C$

d) $\ln|4x^3 - 4x| + C$

e) $\dfrac{1}{(x^4 - 2x^2)^2} + C$

14. A cargo train is moving along a straight track at a rate (in feet/minute) that is given by the equation

$$v(t) = 6t^2\sqrt{343 - t^3} \qquad (0 \le t \le 7)$$

after t minutes. Find the **average** distance the train travels each minute between the 2nd and 6th minutes. (Round to the nearest foot.)

a) 572 feet

b) 2288 feet

c) 6267 feet

d) 1567 feet

e) 1604 feet

15. Find the area underneath the curve $g(x) = 4xe^{-x^2}$ on the interval from $x = 0$ to $x = 2$.

a) 0.147 square units

b) 0.037 square units

c) -0.037 square units

d) 0.293 square units

e) 1.963 square units

16. Albert purchased a new espresso machine for his coffee shop that he expects will generate income at a rate of

$$R(t) = 55,000$$

dollars each year, for the next 7 years. If the income from the machine is reinvested at a rate of 9% per year compounded continuously, find the total accumulated value of this income stream after 7 years, to the nearest dollar.

a) \$622,791
b) \$385,000
c) \$536,318
d) \$285,638
e) \$419,650

17. The demand function for a brand of certain printer ink replacement cartridges is given by

$$p = -0.02x^2 - 0.5x + 14$$

where p is the unit price in dollars, and x is the number of cartridges demanded per week, in thousands. Determine the consumers' surplus if the market price is set at \$11/cartridge.

a) \$62,917
b) \$114,877
c) \$7,917
d) \$6,080
e) \$59,877

18. The economic advisors of a certain country are interested in the wealth distribution of various professions. They find that the Lorentz curve for the distribution of income of dentists was modeled by

$$f(x) = \frac{6}{7}x^2 + \frac{1}{7}x$$

and the distribution of income of accountants was

$$g(x) = \frac{12}{13}x^2 + \frac{1}{13}x$$

Find the coefficient of inequality of dentists, the coefficient of inequality of accountants, and state which distribution is more equitable.

a) 0.2857; 0.3077; accountants
b) 0.2857; 0.3077; dentists
c) 0.1429; 0.1539; dentists
d) 0.5714; 0.4615; dentists
e) 0.5714; 0.4615; accountants

19. Find the indefinite integral

$$\int \left(\frac{1}{x} - 9\right)(x^3 - x)\, dx$$

a) $-\frac{9}{4}x^4 + \frac{1}{3}x^3 + \frac{9}{2}x^2 - x + C$

b) does not exist

c) $-27x^2 + 2x + 9 + C$

d) $(\ln|x| - 9x)(3x^2 - 1) + C$

e) $(\ln|x| - 9x)(x^3 - x) + (\frac{1}{x} - 9)(\frac{1}{4}x^4 - \frac{1}{2}x^2) + C$

20. Sales at the Leibniz Boat Emporium are currently increasing at a rate of

$$5 + 0.5t^{3/2}$$

hundred boats per year, where t is the time in years. It is determined that if they follow a new aggressive advertising plan, their sales will increase at a rate of

$$5e^{0.4t}$$

hundred boats per year. Assuming that the plan is implemented, how many more boats can they expect to sell over the next 4 years than they would have without the new advertisement?

a) 3551 boats

b) 1577 boats

c) 841 boats

d) 2477 boats

e) 2301 boats

Useful Formulas:

$$CS = \int_0^{\overline{x}} D(x)\,dx - \overline{p}\,\overline{x}$$

$$PS = \overline{p}\,\overline{x} - \int_0^{\overline{x}} S(x)\,dx$$

$$A = e^{rT} \int_0^T R(t)e^{-rt}\,dt$$

$$PV = \int_0^T R(t)e^{-rt}\,dt$$

$$A = \frac{mP}{r}(e^{rT} - 1)$$

$$PV = \frac{mP}{r}(1 - e^{-rT})$$

$$L = 2\int_0^1 [x - f(x)]\,dx$$

$$a = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$