

Math 1331 Final Exam Spring 2011

Show all work in blue book! A correct answer with no work is worth 1 point. Please circle all your answers and work all questions in order. Answer all word problems with a complete sentence. Use 4 decimal places for your answers (when needed).

1. The weekly demand for the Pulsar BC color LED television is given by

$$p = \frac{2x - 1}{x^2 - 3x}$$

where p denotes the wholesale unit price in dollars and x denotes the quantity demanded. The weekly total cost function associated with manufacturing the Pulsar BC is given by

$$C(x) = \frac{2x + 4}{x + 3}$$

where $C(x)$ denotes the total cost incurred in producing x sets.

- Find the revenue function R and the profit function P .
 - Find the marginal profit function.
2. Custom Office makes a line of executive desks. It is estimated that the total cost of making x units of their Senior Executive model (in dollars per year) is given by

$$C(x) = 4xe^{-3x^2} + 6x$$

- Find the average cost function.
 - In your own words, explain what happens to the average cost when x is very large?
3. The concentration of a certain drug (in mg/cm²) in a patient's bloodstream t hr after the injection is given by the following function.

$$C(t) = \frac{t^4 + 5t - 6}{t^6 - 3t + 2}$$

How much of the drug remains in the system as time increases without bound? That is, evaluate the limit of the concentration as time, t , goes to infinity. Explain what is happening.

4. The total worldwide box-office receipts for a long-running movie are approximated by the function

$$F(x) = \ln|x^6 + 4x^4 - 3x^2 - 2|$$

where $F(x)$ is measured in millions of dollars and x is the number of years since the movie's release. How fast is the total receipts changing 3 years after its release?

5. The following data shows the relationship how many black bears are in Tennessee in the given year.

Year	1980	1985	1990	1995	2000	2005
Number of Black Bears	504	864	1027	946	1214	1739

- Find the cubic function that best represents the data. (Let $t = 0$ correspond to 1980)
 - Find the point at which the rate of the bears is a maximum. (That is, find the inflection point.)
6. The height of an object thrown in the air is given by $s(t) = e^{-t^2+8}$ where $s(t)$ is given in feet, and t is in seconds. What is the acceleration of the object at 3 seconds?
7. Company J found the following information concerning their profit where x is the number of items sold in hundreds and the profit is in hundreds of dollars. The profit is increasing on the following intervals $(0,5)$ and $(9,\infty)$, the profit is decreasing $(5,9)$. When they sell 500 items, the profit is \$1000. And when they sell 900 items, they lose \$200. Sketch the graph of the profit function. (Note: You can have negative profit.)

8. Lynbrook West, an apartment complex, has 100 two-bedroom units. The monthly profit (in dollars) realized from renting out x apartments is given by

$$P(x) = x^3 - 6x^2 + 40, \quad 0 \leq x \leq 10$$

To maximize the monthly rental profit, how many units should be rented out? What is the maximum monthly profit realizable (i.e. what is the maximum profit)? Make sure you check your endpoints!

9. A 900 square feet rectangular area is to be fenced in along a river. The area is to be fenced in with one side parallel to the river, and two sides perpendicular to it. In addition, the area is to be divided by another fence perpendicular to the river. The three outer fences cost \$4 per foot, and dividing fence cost \$2 per foot. Find the dimensions that minimize the cost of the fence.



10. Evaluate the following integrals.

a.

$$\int_1^2 \frac{3x^2 - 4x}{x^3 - 2x^2 + 3} dx$$

b.

$$\int \left(7x^3 - \frac{3}{x^6} + \sqrt{x} + 1 \right) dx$$

11. Ca rlota Music Company estimates that marginal cost of manufacturing its Christmas series CD is given by the following in dollars/week when the level of production is x CD/week.

$$C'(x) = 0.13x^3 - x + 30$$

The fixed costs incurred by Carlota are \$506/week. Find the total weekly cost $C(x)$ incurred by Carlota in manufacturing x CD/week.

12. Suppose copper is being extracted from a certain mine at a rate given by

$$\frac{d}{dt}P(t) = 100e^{-0.6t}$$

where $P(t)$ is measured in tons of copper and t is measured in years. At this rate how much copper will be extracted in the first 3 years of production?

13. Find the area between two curves $f(x) = -3x^2 + 15$ and $g(x) = 3x^2 - 18x + 15$.

14. The price-supply for a certain product is given by $S(x) = x^2 + 10x + 450$ and the price demand is given by $D(x) = -2x^2 + 850$.

a. Find the equilibrium price and quantity.

b. Find the producers' surplus realized at the equilibrium price.

Formulas for Math 1331

$$\text{Average Value: } \frac{1}{b-a} \int_a^b f(x) dx$$

$$L = 2 \int_0^1 [x - f(x)] dx$$

$$CS = \int_0^{\bar{x}} D(x) dx - \bar{p}\bar{x}$$

$$PS = \bar{p}\bar{x} - \int_0^{\bar{x}} S(x) dx$$

$$A = e^{rT} \int_0^T R(t) e^{-rt} dt$$

$$PV = \int_0^T R(t) e^{-rt} dt$$

$$A = \frac{mP}{r} (e^{rT} - 1)$$

$$PV = \frac{mP}{r} (1 - e^{-rT})$$