MATH 1331, Fall 2012

Final Exam

All problems should be completed in your blue book. Remember to SHOW ALL WORK to receive full credit.

1. The total domestic box office receipts for a particular movie are approximated by

$$T(x) = \frac{120x^2}{x^2+4}$$

where $T(x)$ is measured in millions of dollars and x is the number of months since the movie's release. How much can the movie be expected to make in the long run (that is, as x goes to infinity)?

2. The demand function for a brand of wristwatch is given by

$$d(x) = \frac{40x}{0.1x^2+1}$$

where $d(x)$ is the quantity demanded per week (measured in units of a thousand) and x is the unit price (in dollars).

At what rate is the demand for watches changing when the price per watch is \$6?

3. Find the rate at which the graph of the following function is changing:

$$g(x) = e^{5x+2}x^3$$

4. Barney is an executive for a global company, and he has determined that the total sales of the company, R (in millions of dollars), as it relates to the money spent on advertising, x (in millions of dollars), is given by

$$R(x) = -2x^3 + 15x^2 + 22x + 7$$

Find the company's point of diminishing returns on its advertising spending.

5. Find the slope of the tangent line to the graph of

$$f(x) = \ln\left(\sqrt{x}\right)$$

for all x in the domain of the function.

6. A model rocket is launched into the air such that its height (in feet) at t seconds after launch is given by

$$s(t) = -16t^2 + 55t + 8 \ , \quad 0 \leq t \leq 3.5$$

When does the rocket reach its maximum height? (In other words, at what instant will its **velocity** be zero?)

7. Find the marginal cost function for a microwave manufacturing company if the cost incurred for making x microwaves each week is

$$C(x) = 43x + 62\sqrt{x}$$

8. The fee charged by Marshall's small law firm for each client is based on the number of cases they are working at the time. The **price** to hire the firm at a given time is given by

$$p = -8x + 136$$

where x is the number of cases currently being worked. The total expenses incurred by the firm at that time are given by

$$C(x) = -\frac{1}{3}x^3 - 12x^2 + 316x - 2290$$

How many cases should the agency take at a given time to maximize their **profit**?

9. Government economists of a country determined that the purchase of imported cheese is related to a proposed tax by the formula

$$N(x) = \sqrt{10,000 - 40x - 0.02x^2}$$

where $N(x)$ measures the normal consumption of imported cheese (in tons) when a tax of $x\%$ is imposed on it (note that x is *not* in the decimal form of a percentage in this problem).

Find the rate of change of $N(x)$ when the tax rate is 15%.

10. Find the absolute maximum and absolute minimum of the following function on the interval $[-2, 8]$.

$$f(x) = \frac{1}{3}x^3 + x^2 - 35x + 16$$

11. Ted is finishing the architectural designs on a custom house and wants to include a garden. For monetary reasons, he can only afford to use 50 ft. of fencing to enclose the garden, which is to be rectangular in shape. If he does not need to use fencing along the one side of the garden which touches the house, what are the dimensions of the maximum area that can be enclosed?

12. Robin is starting a talent agency for aspiring Canadian pop stars that she expects will generate income at a rate of

$$R(t) = 385,000$$

dollars each year for 20 years. If the prevailing interest rate over those 20 years is 8% per year compounded continuously, find the present value of the franchise.

13. Maclaren's Pub is considering a new advertising plan which they anticipate will increase their sales at a rate of

$$3e^{0.5t}$$

thousand dollars per year, where t is the time in years. How much more total money can they expect to bring in over the next 3 years using this new plan if the current rate at which their sales are increasing is

$$2 + 0.3t^{4/3}$$

thousand dollars per year?

14. Find the indefinite integral

$$\int (x^2 + 7)(x - 9)\, dx$$

15. A car is moving along a straight road in such a way that it has a velocity (in feet/second) given by

$$v(t) = 4t\sqrt{25 - t^2} \quad (0 \leq t \leq 5)$$

after t seconds. Find the **average** distance the car travels each second between the 1st and 4th seconds.

16. An economist working for a state's development board finds that the Lorentz curve for the distribution of income of college professors was modeled by

$$f(x) = \frac{13}{14}x^2 + \frac{1}{14}x$$

and the distribution of income of newscasters was

$$g(x) = \frac{9}{11}x^4 + \frac{2}{11}x$$

Which profession has a more equitable distribution of income?

17. Find the indefinite integral

$$\int x^2 e^{-x^3}\, dx$$

18. Lily owns a company that publishes activity books for elementary school children and she determines that the supply function for her company is given by

$$p = 5 + \frac{1}{2}x^2$$

where p is the unit price in dollars, and x is the number of books supplied, in thousands, per month. Determine the producers' surplus when the market unit price is set at \$23/book.

19. Find the area underneath the curve
$$\int \frac{x^2 - 1}{x^3 - 3x}\, dx$$
on the interval from $x = 2$ to $x = 4$.

Useful Formulas:

$$CS = \int_0^{\overline{x}} D(x)\,dx - \overline{p}\,\overline{x}$$

$$PS = \overline{p}\,\overline{x} - \int_0^{\overline{x}} S(x)\,dx$$

$$A = e^{rT} \int_0^T R(t)e^{-rt}\,dt$$

$$PV = \int_0^T R(t)e^{-rt}\,dt$$

$$A = \frac{mP}{r}(e^{rT} - 1)$$

$$PV = \frac{mP}{r}(1 - e^{-rT})$$

$$L = 2\int_0^1 [x - f(x)]\,dx$$