1331 Final Exam, Spring 2013

You must show your work, and the work you show must yield the answer you obtain, if you are to receive credit. Present the problems in your blue book in the order that they occur on the exam: problem 1 first, problem 2 second, etc.

**Derivative formulas**

$$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx} \qquad \frac{d}{dx}e^u = e^u\frac{du}{dx} \qquad \frac{d}{dx}\ln(u) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx} \qquad \frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

**Integral formulas**

$$\int u^n\frac{du}{dx}dx = \frac{u^{n+1}}{n+1} + C \qquad \int\frac{1}{u}\frac{du}{dx}dx = \ln|u| + C \qquad \int e^u\frac{du}{dx}dx = e^u + C$$

1. The total worldwide box-office receipts for a movie are approximated by the function
$$T(x) = \frac{110x^2}{x^2+4}.$$
where $T(x)$ is measured in millions of dollars and $x$ is the number of months since the movie's release.
   a. (5pt) What were the total box-office receipts for the first 2 months?
   b. (5pt) What were the box-office receipts for the third month alone?
   c. (5pt) What was the rate of change of the receipts per month on the last day of the third month?
   d. (5pt) What is anticipated to be the total box-office receipts for the life of the movie.
2. The demand for Sportsman 5 X 7 tents is given by
$$p = f(x) = -0.1x^2 - x + 50.$$
where $p$ is measured in dollars and $x$ is measured in units of 1000.
   a. (5pt) Find the average rate of change in the unit price of a tent if the quantity demanded grows from 5000 to 5100 tents.
   b. (5pt) What is the instantaneous rate of change at the demand level of 5050 tents?
   c. Show that $p$ is a decreasing function of the demand. (Note that demand $x \geq 0$.)
   d. (5pt) What is the revenue from the sale of 5000 tents?

3. An efficiency study showed that number of radios assembled by an average worker during the morning shift $t$ hours after starting work at 8 A.M. is given by
$$N(t) = -t^3 + 5t^2 + 15t \qquad (0 \le t \le 4)$$
   a. (5pt) Find the rate at which the average worker will be assembling radios $t$ hours after starting work.
   b. (5pt) At what rate will the average worker be assembling radios at 11 A.M.?
   c. (5pt) Find the interval in which $N''(t) < 0$.
   d. (5pt) Interpret the result in part (c).
4. A company manufactures television sets. The quantity $x$ of these sets (in thousands) demanded each week is related to the wholesale unit price $p$ by the following equation:
$$p(x) = 700 - 25x.$$
   The weekly total cost incurred for producing $x$ sets is given by $C(x) = 50x + 450$.
   a. (4pt) Find the average cost function.
   b. (4pt) Find the revenue function $R(x)$.
   c. (4pt) Find the marginal revenue function.
   d. (4pt) Find the profit function $P(x)$.
   e. (4pt) Find the marginal profit function.
5. The monthly demand, $x$, of a compact disc is related to the price, $p$ dollars per disc by the following equation:
$$p(x) = -0.0004x + 6.$$
   The total monthly cost for pressing and packaging $x$ copies is given by
$$C(x) = 600 + 2x - 0.0002x^2.$$
   a. (5pt) What is the production level that minimizes the total cost?
   b. (5pt) What is the revenue function?
   c. (5pt) What is the profit function?
   d. (5pt) What is the production level that maximizes the profit?
6. The owner of a luxury motor yacht charges \$500/person/day if exactly 15 people show up for the cruise. However, if more than 15 people sign everyone's fare is reduced by \$3 for each additional passenger. Assume that at least 15 people sign up for the cruise.
   a. (5pt) Find the revenue function.
   b. (5pt) What is the number of people that will result in the maximum revenue?
   c. (5pt) What is the maximum revenue?
   d. (5pt) What is the fare per person in this case?
7. A company makes electronic flashes for cameras. The estimated marginal profit associated with producing and selling $x$ flashes per month is $P'(x) = -0.005x + 25$. The fixed cost for producing and selling these flashes is \$15,000/month.
   a. (5pt) What is the monthly production level that maximizes the profit?
   b. (10pt) Find the profit function $P(x)$.
   c. (5pt) Compute the maximum profit.

8. It is estimated that a newly discovered oil field can be expected to produce oil at the rate of

$$R(t) = \frac{600t^2}{t^3 + 32} + 5 \qquad (0 \le t \le 20)$$

thousand barrels/year, $t$ years after production begins.

   a. (10pt) Find the formula for the number of barrels produced in $t$ years.
   b. (10pt) Find the expected yield during the first 5 years.

9. A toy company has determined that the quantity **demanded** $x$ of their product in units of a thousand is related to the unit price $p$ dollars by the relation $p(x) = 144 - x^2$. The company will **supply** x units of its product if the market unit price is $p(x) = 48 + \frac{1}{2}x^2$.
   a. (5pt) Determine the unit quantity $x$ for which supply equals demand.
   b. (5pt) Determine the equilibrium price.
   c. (5pt) Determine the consumer's surplus at the equilibrium price.
   d. (5pt) Determine the producer's surplus at the equilibrium price

10. A study has concluded that the Lorenz curve for the distribution of income of college teachers is given by the function

$$f(x) = \frac{13}{14}x^2 + \frac{1}{15}x,$$

and that of lawyers is given by the function

$$g(x) = \frac{9}{11}x^2 + \frac{2}{11}x.$$

   a. (5pt) Compute the coefficient of inequality for teachers.
   b. (5pt) Compute the coefficient of inequality for lawyers.
   c. (5pt) Which profession has the more equitable income distribution.
   d. (5pt) Justify your answer in part (c).