Final Exam, Form A
MATH 1331-24, Fall 2014

Print Name:____

For full credit, neatly show your work.

You may use any TI 83 or TI84 series calculator. You may NOT use a TI 89, TInspire, TInspire with an 84 faceplate or Casio/HP equivalents, your cell phone or anything capable of storing information or connecting with the internet.

On this exam, no books, notes or notecards are allowed. There is a formula sheet you may carefully detach from the back of this exam.

(3 pts) 1. The revenue from manufacturing x items is given by

$$R(x)=\frac{8x^4-3x^2-5}{4x^4-x^3+2}$$

where x is given in hundreds of units and the revenue is in hundreds of dollars. Find the revenue as the number of units increases, $x\to\infty$.

(3 pts) 2. Given the cost function for producing a product, $C(x)=7x^3-0.2x^2+5x+30$, find the average cost function, $\overline{C}(x)$.

(3 pts) 3. Suppose that the profit (in hundreds of dollars) from selling x units of a product is given by $P(x) = \frac{x^2}{2x+1}$. Find the marginal profit.

(3 pts) 4. The sales of a company are related to its expenditures on research by $S(x) = 7x^2e^{-3x}$. Find $\frac{dS}{dx}$.

(3 pts) 5. If the total revenue received from the sale of x items is given by $R(x) = 30\ln(2x+1)$, find the marginal revenue.

(4 pts) 6. Assume the demand function is given by $q = 5000 - 100p$.

a. Find the revenue function. Solve the demand equation for p and use $R(q) = qp$.

b. Find the marginal revenue function.

c. Find the marginal revenue for a production level of 400 units.

(4 pts) 7. A manufacturer sells a product with the following cost and revenue functions (in dollars), where x is the number of items sold, for $0 \le x \le 900$.

$$P(x) = 4.73x - 0.00473x^2$$

a. At what production levels is the profit function increasing?

b. What is the maximum profit?

(6 pts) 8. Find the point of diminishing returns *(x, y)* for $R(x) = 10,000 - x^3 + 27x^2 + 600x, \quad 0 \le x \le 20$, where *R(x)* represents revenue in thousands of dollars and *x* represents the amount spent on advertising in tens of thousands of dollars.

(7 pts) 9. ECONOMIC ORDER QUANTITY Suppose 420,000 cases of barbecue sauce are to be manufactured annually. It costs \$2 per year for electricity to store a case and \$50 to produce each batch. How many production runs should there be each year to minimize costs? Round to the nearest case and production run.

(7 pts) 10. A study of the demand for Serious Satellite Radio Systems in Lubbock found that the demand depends on the discount according to the equation $q = 25{,}000 - 50p$.

a. Determine the elasticity of demand, E.

b. Is the demand for radios elastic, inelastic or neither when the price is $150?

c. Find the price that maximizes revenue.

(7 pts) 11. Given the revenue and cost functions $R(x) = 32x - 0.3x^2$ and $C(x) = 4x + 10$, where x is the daily production,

a. find the profit function.

b. Find the rate of change of profit **with respect to time** when 15 units are produced and the rate of change of production is 6 units per day.

(8 pts) 12. Suppose the supply function of a certain item is given by $S(q) = 4q + 2$ and the demand function is given by $D(q) = 14 - q^2$.

a. Find the equilibrium price.

b. Find the producers' surplus at the equilibrium price. **Set up and simplify the integral. Do not solve.**

(9 pts) 13. For the given marginal revenue function, $R'(x) = \dfrac{-10q^3}{5q^4 + 2000}$, find:

a. the revenue function. Remember, if no items are sold, the revenue is 0, $R(0) = 0$. Round the constant to one decimal place.

b. the demand function.

(6 pts) 14. Otis Taylor plots the price per share of a stock that he owns as a function of time and finds that it can be approximated by the function

$$S(t) = 37 + 6e^{-0.03t} \quad ,$$

where t is the time (in years) since the stock was purchased. Find the average price of the stock over the first four years. **Set up and simplify the integral only. Do not solve.**

(8 pts) 15. Suppose a company wants to introduce a new machine that will produce a rate of annual savings (in dollars) given by the function, $S'(t) = -t^2 + 7t + 4$, where x is the number of years of operation of the machine, while producing a rate of annual costs (in dollars) given by the function , $C'(t) = t^2$.

a. For how many years will it be profitable to use this new machine?

b. What are the net total savings over the entire period of use of the machine? **Set up and simplify the integral only. Do not solve.**

(7 pts) 16. The rate of change of revenue (in dollars per calculator) from the sale of x calculators is $R'(x) = xe^{2x}$. Find the total revenue from the sale of the first 2 calculators.

(12 pts) 17. If money is flowing continuously at a constant rate of \$3000 per year over 5 years at 6% interest compounded continuously, find:

a. the total money flow over the 5-year period.

b. the present value of the amount with interest.

c. the accumulated amount of money flow, compounded continuously, at time T = 5.

d. the total interest earned.

Formulas

$$E = -\frac{p}{q} \cdot \frac{dq}{dp}$$

$$\frac{dR}{dp} = q(1 - E)$$

$$q = \sqrt{\frac{2fM}{k}}$$

$$\text{Consumers' Surplus} = \int_0^{q_0} \left[D(q) - p_0\right] dq$$

$$\text{Producers' Surplus} = \int_0^{q_0} \left[p_0 - S(q)\right] dq$$

$$f(t) = Ce^{kt}$$

$$\text{Total Money Flow} = \int_0^T f(t)\,dt$$

$$\text{Present Value of Money Flow} = \int_0^T f(t)e^{-rt}\,dt$$

$$\text{Accumulated Amount of Money Flow} = e^{rT}\int_0^T f(t)e^{-rt}\,dt$$

$$\text{Average Value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\int u\,dv = uv - \int v\,du$$