1. The revenue from producing Weasley’s Wildfire Whiz-Bangs is given by

\[ R(x) = \frac{8x^7 - 6x^5 + 9x^4 + 10}{6x^7 + 9x^6 - 3x^3 + x - 7} \]

where \( x \) is given in hundreds of units and the revenue is in hundreds of dollars. Find the revenue as the number of units increases, as \( x \to \infty \). Explain the meaning of this limit in a complete sentence. Round to the nearest cent.

2. Find \( k \) so that the cost function, \( C(x) \), is continuous on \( (-\infty, \infty) \).

\[ C(x) = \begin{cases} 
\frac{x^2 - 3x - 10}{x - 5}, & \text{if } x < 5 \\
\frac{x - 5}{3x + k}, & \text{if } x \geq 5 
\end{cases} \]

3. Find the marginal profit for each of the profit functions.
   a) \( P(x) = 5x^{-4} + 3\sqrt{x} \)
   b) \( P(x) = (x + 6x^{-2})e^{x^2} \)

4. Find the equation of the line tangent to \( f(x) = 2\ln(x + 6) \) at \((-5,0)\). Leave \( e \) and \( \ln \) in your answer. Do NOT use decimals in your answer.

5. A cost function is given by

\[ C(x) = \frac{e^{x^2+4x}}{x + 4} \]

a) Find the average cost function, \( \bar{C}(x) = \frac{C(x)}{x} \).

b) Find the marginal average cost function.

6. Assume the demand function for a certain commodity has the form

\[ p = x^4 - 7x^3 + 3x^2 - 7 \]

where \( x \) is the quantity demanded, measured in units of thousands and \( p \) is the unit price is dollars.

a) Find the revenue function, \( R(x) \).

b) Find the marginal revenue function.

c) Calculate \( R'(3) \).

7. The concentration (in milligrams/cubic centimeters) of a certain drug in a patient’s body \( t \) hr after infection is given by

\[ C(t) = \frac{t^3}{3} - 6t^2 + 35t + 600 \]

When is the concentration of the drug increasing, and when is it decreasing?
8. A certain company wants to investigate the effects of advertisement on the revenue of their product. The following function is used to model the situation:

\[ R(x) = \frac{3x^5}{4} - 10x^3 \]

where \( R \) is the monthly revenue in thousands of dollars, and \( x \) is the amount of advertising in thousands of dollars each month. Find the intervals where the revenue is concave up and concave down. State the inflection points.

9. A company has found the following information about their profit.

- Relative maximum is \((2, 25)\)
- Relative minimum is \((-2, -7)\)
- Increasing on \((-2, 2)\)
- Decreasing on \((-\infty, -2) \cup (2, \infty)\)
- Inflection Point is \((0, 9)\)
- Concave up on \((-\infty, 0)\)
- Concave down on \((0, \infty)\)

Graph the profit function.

10. For a certain company, the revenue in thousands of dollars when \( x \) in hundreds of units are sold is given by

\[ R(x) = x^3 + 6x^2 - 32x + 200. \]

For the same company, the cost in thousands of dollars when \( x \) units are sold is given by

\[ C(x) = 21x^2 - 104x + 55. \]

a) Find the profit equation.
b) How many items does the company need to sale to maximize the profit? What is the maximum profit?

*Answer in complete sentences.*

11. A rectangular box is to have a square base and a volume of 64 ft\(^3\). If the material for the base cost $3 per square foot, the material for the sides costs $2 per square foot, and the material for the top costs $1 per square foot, determine the dimensions of the box that can be constructed at minimum cost. What is the minimum cost?

*Answer in complete sentences.*

12. Integrate the following:

a) \[
\int \left( x^4 - x^{-7} + \sqrt{x} - 1 \right) \, dx
\]

b) \[
\int \left( 7x^6 - 12x^3 + 5 \right) \left( x^7 - 3x^4 + 5x \right)^3 \, dx
\]

c) \[
\int_{0}^{2} \frac{-8x - 12}{(x^2 + 3x - 6)^4} \, dx
\]

13. The marginal revenue for a certain company is given by

\[ R'(x) = 3x^4 + 3x^2 - x - 7 \]

Find the revenue function, keeping in mind that \( R(0) = 0 \). Find the demand function. *Clearly label both formulas.*
14. The marginal cost function for a certain company is given by
\[ C'(x) = \frac{4x^3 - 6x}{x^4 - 3x^2 + 5} \]
Find the cost function where the fixed cost is 50 \( \ln 5 \); that is, \( C(0) = 50 \ln 5 \).

15. Find the area between \( f(x) = 3x^3 \) and \( g(x) = 3x \). Make sure to graph the region. Make sure to include units in your answer.

16. A supplier of portable hair dryers will make \( x \) hundred units of hair dryers available in the market when the unit price is
\[ p = \sqrt{8 + 4x} = S(x) \]
dollars. Determine the producers’ surplus if the market price is set at $6/unit. Answer in a complete sentence.

17. Camille purchased a 21-year franchise for a computer outlet store that is expected to generate income at the rate of
\[ R(t) = 350,000 \] dollars/year. If the prevailing interest rate is 6%/year compounded continuously, find the present value of the franchise. Round to the nearest cent.

18. The revenue in thousands of dollars for a company that makes two different products is given by
\[ R(x, y) = x^2 - y^2 + 6x + 7y - 8 \]
Find \( R(12, 6) \). Answer in a complete sentence.

19. The monthly profit (in dollars) of Bond and Barker Department Store depends on the level of inventory \( x \) (in thousands of dollars) and the floor space \( y \) (in thousands of square feet) available for display of the merchandise, as given by the equation.
\[ P(x, y) = -5x^2 - 7y^2 + xy + 25x + 30y - 30,000 \]
Compute \( \frac{\partial P}{\partial x} \) and \( \frac{\partial P}{\partial y} \).

Formulas for Math 1331

\[
L = 2 \int_0^1 [x - f(x)] \, dx \quad \quad CS = \int_0^x D(x) \, dx - \bar{p} \cdot \bar{x} \quad \quad PS = \bar{p} \cdot \bar{x} - \int_0^x S(x) \, dx
\]
\[ A = e^{rT} \int_0^T R(t)e^{-rt} \, dt \quad \quad PV = \int_0^T R(t)e^{-rt} \, dt \quad \quad A = \frac{mP}{r} (e^{rT} - 1) \]
\[ PV = \frac{mP}{r} (1 - e^{-rT}) \]