Math 1331 **Final Exam** **Spring 2014**

Directions: Work all questions in order and label the questions. **Show all work. Box answers.**

1. The revenue from producing Weasley's Wildfire Whiz-Bangs is given by
$$R(x) = \frac{8x^7 - 6x^5 + 9x^4 + 10}{6x^7 + 9x^6 - 3x^3 + x - 7}$$
where x is given in hundreds of units and the revenue is in hundreds of dollars. Find the revenue as the number of units increases, as $x \to \infty$. Explain the meaning of this limit in a complete sentence. *Round to the nearest cent.*

2. Find k so that the cost function, $C(x)$, is continuous on $(-\infty, \infty)$.
$$C(x) = \begin{cases} \dfrac{x^2 - 3x - 10}{x - 5}, & \text{if } x < 5 \\ \dfrac{3x + k}{2}, & \text{if } x \geq 5 \end{cases}$$

3. Find the marginal profit for each of the profit functions.
   a) $P(x) = 5x^{-4} + 3\sqrt{x}$
   b) $P(x) = (x + 6x^{-2})e^{x^2}$

4. Find the equation of the line tangent to $f(x) = 2\ln(x + 6)$ at $(-5, 0)$. *Leave e and* ln *in your answer. Do NOT use decimals in your answer.*

5. A cost function is given by
$$C(x) = \frac{e^{x^2 + 4x}}{x + 4}$$
   a) Find the average cost function, $\bar{C}(x) = \frac{C(x)}{x}$.
   b) Find the marginal average cost function.

6. Assume the demand function for a certain commodity has the form
$$p = x^4 - 7x^3 + 3x^2 - 7$$
where x is the quantity demanded, measured in units of thousands and p is the unit price is dollars.
   a) Find the revenue function, $R(x)$.
   b) Find the marginal revenue function.
   c) Calculate $R'(3)$.

7. The concentration (in milligrams/cubic centimeters) of a certain drug in a patient's body t hr after infection is given by
$$C(t) = \frac{t^3}{3} - 6t^2 + 35t + 600$$
When is the concentration of the drug increasing, and when is it decreasing?

8. A certain company wants to investigate the effects of advertisement on the revenue of their product. The following function is used to model the situation:

$$R(x) = \frac{3x^5}{4} - 10x^3$$

where R is the monthly revenue in thousands of dollars, and x is the amount of advertising in thousands of dollars each month. Find the intervals where the revenue is concave up and concave down. State the inflection points.

9. A company has found the following information about their profit.

   Relative maximum is $(2, 25)$
   Relative minimum is $(-2, -7)$
   Increasing on $(-2, 2)$
   Decreasing on $(-\infty, -2) \cup (2, \infty)$
   Inflection Point is $(0, 9)$
   Concave up on $(-\infty, 0)$
   Concave down on $(0, \infty)$

   Graph the profit function.

10. For a certain company, the revenue in thousands of dollars when x in hundreds of units are sold is given by

$$R(x) = x^3 + 6x^2 - 32x + 200.$$

For the same company, the cost in thousands of dollars when x units are sold is given by

$$C(x) = 21x^2 - 104x + 55.$$

   a) Find the profit equation.
   b) How many items does the company need to sale to maximize the profit? What is the maximum profit? *Answer in complete sentences.*

11. A rectangular box is to have a square base and a volume of 64 ft^3. If the material for the base cost \$3 per square foot, the material for the sides costs \$2 per square foot, and the material for the top costs \$1 per square foot, determine the dimensions of the box that can be constructed at minimum cost. What is the minimum cost? *Answer in complete sentences.*

12. Integrate the following:

   a)
$$\int \left(x^4 - x^{-7} + \sqrt{x} - 1\right)\, dx$$

   b)
$$\int (7x^6 - 12x^3 + 5)(x^7 - 3x^4 + 5x)^3\, dx$$

   c)
$$\int_0^2 \frac{-8x - 12}{(x^2 + 3x - 6)^4}\, dx$$

13. The marginal revenue for a certain company is given by

$$R'(x) = 3x^4 + 3x^2 - x - 7$$

Find the revenue function, keeping in mind that $R(0) = 0$. Find the demand function. *Clearly label both formulas.*

14. The marginal cost function for a certain company is given by

$$C'(x) = \frac{4x^3 - 6x}{x^4 - 3x^2 + 5}$$

Find the cost function where the fixed cost is $50 \ln 5$; that is, $C(0) = 50 \ln 5$.

15. Find the area between $f(x) = 3x^3$ and $g(x) = 3x$. *Make sure to graph the region. Make sure to include units in your answer.*

16. A supplier of portable hair dryers will make x hundred units of hair dryers available in the market when the unit price is

$$p = \sqrt{8 + 4x} = S(x)$$

dollars. Determine the producers' surplus if the market price is set at $6/unit. *Answer in a complete sentence.*

17. Camille purchased a 21-year franchise for a computer outlet store that is expected to generate income at the rate of

$$R(t) = 350{,}000$$

dollars/year. If the prevailing interest rate is 6%/year compounded continuously, find the present value of the franchise. *Round to the nearest cent.*

18. The revenue in thousands of dollars for a company that makes two different products is given by

$$R(x, y) = x^2 - y^2 + 6x + 7y - 8$$

Find $R(12, 6)$. *Answer in a complete sentence.*

19. The monthly profit (in dollars) of Bond and Barker Department Store depends on the level of inventory *x* (in thousands of dollars) and the floor space *y* (in thousands of square feet) available for display of the merchandise, as given by the equation.

$$P(x, y) = -5x^2 - 7y^2 + xy + 25x + 30y - 30{,}000$$

Compute $\frac{\partial P}{\partial x}$ and $\frac{\partial P}{\partial y}$.

Formulas for Math 1331

$$L = 2\int_0^1 [x - f(x)]\ dx$$

$$CS = \int_0^{\bar{x}} D(x)\ dx - \bar{p} * \bar{x}$$

$$PS = \bar{p} * \bar{x} - \int_0^{\bar{x}} S(x)\ dx$$

$$A = e^{rT}\int_0^T R(t)e^{-rT}\ dt$$

$$PV = \int_0^T R(t)e^{-rt}\ dt$$

$$A = \frac{mP}{r}\,(e^{rT} - 1)$$

$$PV = \frac{mP}{r}(1 - e^{-rT})$$