

1331 Final Exam, Fall 2015

You must show your work, and the work you show must yield the answer you obtain, if you are to receive credit. Present the problems in your blue book in the order that they occur on the exam: problem 1 first, problem 2 second, etc. Allow at least one full page for each problem.

Derivative formulas

$$\frac{d}{dx} u^n = n u^{n-1} \frac{du}{dx}$$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

$$\frac{d}{dx} \ln(u) = \frac{1}{u} \frac{du}{dx}$$

$$\frac{d}{dx} uv = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d}{dx} \frac{u}{v} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Integral formulas

$$\int u^n \frac{du}{dx} dx = \frac{u^{n+1}}{n+1} + C \text{ for } x \neq 1$$

$$\int \frac{1}{u} \frac{du}{dx} dx = \ln|u| + C$$

$$\int e^u \frac{du}{dx} dx = e^u + C$$

1. (20p) Basic Skills (Derivatives): Please compute the following derivatives

a. (4p) $\frac{d}{dx} (x^2 + x)^5$

b. (4p) $\frac{d}{dx} \frac{1}{\sqrt{3x+1}}$

c. (4p) $\frac{d}{dx} e^{3x-2}$

d. (4p) $\frac{d}{dx} \ln(x^3 - 2x)$

e. (4p) $\frac{d}{dx} (x^2 + 1)(x - 1)^6$

2. (20p) Basic Skills (Integrals): Please compute the following indefinite integrals

a. (4p) $\int x + \frac{1}{x} dx$

b. (4p) $\int \frac{x^4 + x}{x^3} dx$

c. (4p) $\int \sqrt{5x+2} dx$

d. (4p) $\int e^{3x-2} dx$

e. (4p) $\int \frac{x}{3x^2 + 1} dx$

3. (20p) Piecewise Function: In a new promotional ad campaign, a rental car company has just announced that it will charge \$25 per day, or portion of a day, to rent a full sized car for a period of 1 to 5 days with days 6 and 7 for free.

- (4p) How much does it cost to rent the car for 4 days?
- (4p) How much does it cost to rent the car for 10 days?
- (4p) Let $C(x)$ represent the cost of using the rental car for x days. (Notice that x can be any portion of a day or days.) Provide a neat sketch of the graph of $C(x)$, on the interval $0 < x \leq 10$.
- (4p) What is the average cost per day if you rent the car for 4 days?
- (4p) What is the average cost per day if you rent the car for 10 days?

4. (20p) Rate of Change: Suppose the revenue (in thousands of dollars) from producing x units is given by the formula $R(x) = 10x - 0.002x^2$.

- (4p) What is the revenue from the production of 100 units?
- (4p) What is the average rate of change of revenue when production changed from 100 units to 101 units?
- (4p) What is the formula for the instantaneous rate of change of revenue with respect to the number of units produced when x units are produced?
- (4p) What is the instantaneous rate of change with respect to the number of units produced when 100 units are produced.
- (4p) How does this number compare with your answer in part b?

5. (20p) Average Cost: Suppose the cost of producing x units of a given product is given by $C(x) = 5 \ln(2x + 1)$.

- (4p) What is the average cost function $\bar{C}(x)$?
- (4p) What is the average cost per unit at the production level of 9 units?
- (4p) What is the marginal average cost function?
- (4p) What is the marginal average cost at the production level of 9 units?
- (4p) Interpret your answer.

6. (20p) Profit to Marginal Profit: A company's costs and revenues in dollars for producing x units of a product are given by $C(x) = 2x$ and $R(x) = 6x - \frac{x^2}{1000}$ respectively.

- (4p) Find the marginal cost and marginal revenue functions.
- (4p) Determine the profit function, $P(x)$.
- (4p) Determine the marginal profit function.
- (4p) For what value of x is the marginal profit equal to 0?
- (4p) Find the profit when the marginal profit is 0?

7. (20p) Maximum Profit: In planning a new restaurant, it is estimated that a profit of \$9 per seat will be made if the number of seats is less than or equal to 50. Due to extra expenses, it is estimated that the profit per seat for all seats will decrease by \$0.10 for every additional seat above 50.

- (4p) What is the estimated profit if restaurant has 50 seats?
- (4p) What is the estimated profit if the restaurant has 60 seats?
- (4p) Let $P(x)$ represent the estimated profit if the restaurant has x seats. Provide the formula for $P(x)$.
- (4p) Find the number of seats that will produce the maximum profit.
- (4p) Find the maximum profit.

8. (20p) Consumer and Producer Surplus: Suppose the supply function for oil is given (in dollars) by $S(q) = q + 1$, and the demand function is given (in dollars) by $D(q) = 9 - q^2$.

- (4p) Graph the supply and demand functions on the same coordinate axis.
- (4p) Find the point at which supply and demand are in equilibrium.
- (4p) Shade in the region that represents the consumers' surplus.
- (4p) Find the consumers' surplus.
- (4p) Find the producers' surplus.

9. (20p) Marginal Profit to Profit: The marginal profit in dollars on Brie cheese sold at a cheese store is given by $P'(x) = 50x^3 + 30x^2$, where x is the amount of cheese sold in **hundreds of pounds**. The “profit” is -\$40 when no cheese is sold.

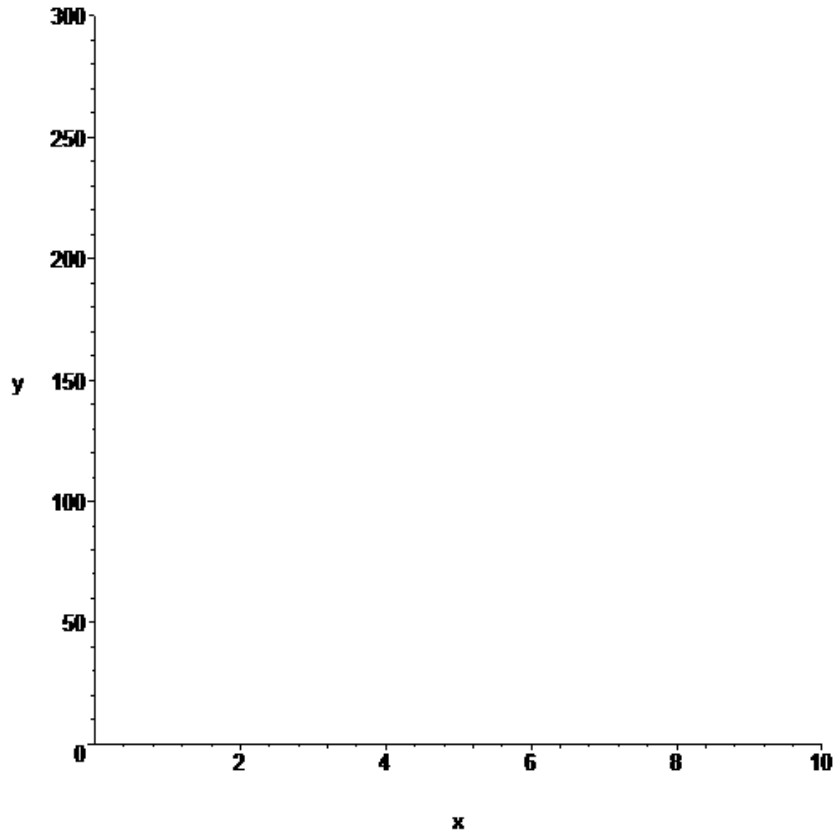
- a. (4p) What is the marginal profit at the sell level of 100 pounds?
- b. (4p) Find the profit function, $P(x)$.
- c. (4p) What is the profit from the sale of 200 pounds?
- d. (4p) What is the profit from the sale of 201 pounds?
- e. (4p) How does this compare to the marginal cost **per pound** at the sell level of 200 pounds?

10. (20p) Inventory (Minimum Cost): A manufacturer has a steady annual demand for 13,950 cases of sugar. It costs \$9 to store one case for one year, \$31 in setup cost to produce each batch, and \$16 to produce each case.

- a. (4p) What is the cost for producing one batch of 13,950 cases (ignoring the storage cost)?
- b. (4p) What would be the storage cost for the year if one batch of 13,950 cases were placed in storage at the first of the year? (Since the demand is steady, we can assume that the average number of cases in storage throughout the year is $\frac{13,950}{2}$.)
- c. (4p) Suppose multiple batches are made with n cases in each batch. How many batches are made?
- d. (4p) What is the total annual cost including storage as a function of n ?
- e. (4p) What is the value of n that yields minimum cost?

Student Name: _____.

Coordinate axis for Problem 3.



Coordinate axis for Problem 8

