

1331 Final Exam, Spring 2016

You must show your work, and the work you show must yield the answer you obtain, if you are to receive credit. Present the problems in your blue book in the order that they occur on the exam: problem 1 first, problem 2 second, etc. Allow at least one full page for each problem.

Derivative formulas

$$\frac{d}{dx} u^n = n u^{n-1} \frac{du}{dx}$$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

$$\frac{d}{dx} \ln(u) = \frac{1}{u} \frac{du}{dx}$$

$$\frac{d}{dx} uv = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d}{dx} \frac{u}{v} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Integral formulas

$$\int u^n \frac{du}{dx} dx = \frac{u^{n+1}}{n+1} + C \text{ for } n \neq -1$$

$$\int \frac{1}{u} \frac{du}{dx} dx = \ln|u| + C$$

$$\int e^u \frac{du}{dx} dx = e^u + C$$

1. (20p) Basic Skills (Derivatives): Please compute the following derivatives

a. (4p) $\frac{d}{dx} (x^2 - x + 4)^5$

b. (4p) $\frac{d}{dx} \sqrt{3x^2 + x + 1}$

c. (4p) $\frac{d}{dx} x e^{x^2}$

d. (4p) $\frac{d}{dx} \ln(x^3 - 2x)$

e. (4p) $\frac{d}{dx} \frac{2x-1}{3x+3}$

2. (20p) Basic Skills (Integrals): Please compute the following indefinite integrals

a. (4p) $\int (x^2 + \frac{1}{x^2} - 5) dx$

b. (4p) $\int (2x^3 + x)^2 dx$

c. (4p) $\int \frac{5}{\sqrt{5x+2}} dx$

d. (4p) $\int x e^{3x^2-2} dx$

e. (4p) $\int \frac{x^3}{3x^4 + 1} dx$

3. (20p) Piecewise Function: A company charges \$1.25 per lb. for a certain fertilizer on all orders 100 lbs. or less, and \$1.00 per lb. for each lb. above 100 lbs.

- (4p) What is the cost of 75 lbs.?
- (4p) What is the cost of 125 lb.?
- (4p) what is the average cost per lb. for an order of 75lb?
- (4p) What is the average cost per lb. for an order of 125 lb.?
- (4p) How many lbs. can you purchase with \$200?

4. (20p) Rate of Change: Suppose customers in a hardware store are willing to buy $N(p)$ boxes of nails at p dollars per box, as given by

$$N(p) = 80 - 5p^2.$$

- (4p) Find the instantaneous rate of change of demand.
- (4p) As the price per box increases, the demand should go down. Is this consistent with the answer you got in part a? Explain.
- (4p) Find the revenue function, $R(p)$.
- (4p) Find the marginal revenue function, $R'(p)$.
- (4p) Find the price (value of p) that yields the maximum revenue.

5. (20p) Average Cost: Suppose the cost of producing x units of a given product is given by

$$C(x) = 81x^2 + 17x + 324.$$

- (4p) What is the average cost function $\bar{C}(x)$?
- (4p) What is the marginal average cost function $\bar{C}'(x)$?
- (4p) What are the three critical points of $\bar{C}(x)$?
- (4p) What is the minimum value of the average cost function on the interval $1 \leq x \leq 10$?
- (4p) What is the minimum value of the average cost function on the interval $10 \leq x \leq 20$?

6. (20p) Marginal Profit: Suppose the cost in dollars of manufacturing q items is given by

$$C(q) = 2000q + 3500,$$

and the demand equation is given by

$$q = \sqrt{15,000 - 1.5p}.$$

- (4p) Solve for p as a function of q , that is find the function $p = D(q)$.
- (4p) Find the revenue function $R(q)$.
- (4p) Find the profit function $P(q)$.
- (4p) Find the marginal profit function $P'(q)$.
- (4p) What is the value of q that makes the marginal profit equal to 0?

7. (20p) Maximum Revenue: The TTU Alumni Association is arranging a charter flight to, and a week in, Cuba. The cost of the trip is \$2500 each for 100 passengers, with a refund of \$10 per passenger for each passenger in excess of 100.

- (4p) What is the cost per passenger if 120 people sign up?
- (4p) What is the revenue for the charter company if 120 passengers sign up?
- (4p) Let $R(x)$ represent the total revenue if x passengers sign up. Find the function $R(x)$.
- (4p) Find the number of seats that will produce the maximum revenue.
- (4p) Find the maximum revenue.

8. (20p) Consumer and Producer Surplus: Suppose the supply function for oil is given (in dollars) by

$$S(q) = q^2 + 1, \text{ and the demand function is given (in dollars) by } D(q) = -q + 7$$

- (4p) Graph the supply and demand functions on the same coordinate axis. Use the graph window with $0 \leq q \leq 3$ for horizontal axis and $0 \leq y \leq 10$ for the vertical axis.
- (4p) Shade in the region that represents the consumers' surplus.
- (4p) Find the point at which supply and demand are in equilibrium.
- (4p) Find the consumers' surplus.
- (4p) Find the producers' surplus.

9. . (20) Marginal Cost to Cost: A company has found that the marginal cost (in thousands of dollars) to produce x central air conditioning units is

$$C'(x) = \frac{60x}{5x^2 + e}$$

where x is the number of units produced.

- (4p) What is the marginal cost at the production level of 100 units?
- (4p) Find the cost function, $C(x)$, given that the company incurs a fixed cost of \$10,000 even if no units are built.
- (4p) What is the total cost for producing 100 air conditioning units?
- (4p) What is the cost attributed to the production of the 101st unit?
- (4p) How does this compare to the marginal cost at the production level of 100 units?

10. (20p) Lot Size (Minimum Cost): A bookstore chain has a steady annual demand for 100,000 copies of a best-selling book. Each copy costs the bookstore \$2, it costs \$0.50 to store 1 copy for 1 year, and it costs \$60 to place an order.

- (4p) What would be the total cost for the year, if the bookstore chain ordered all the books at the first of the year? (Since the demand is steady, we can assume that the average number of cases in storage throughout the year is $\frac{100,000}{2}$.)
- (4) What would be the total cost for the year if the bookstore chain placed 4 equally spaced orders throughout the year? (Notice that the size of each order would be $\frac{100,000}{4} = 25,000$ books.)
- (4p) Suppose equally spaced orders are made with q books in each order. Let n represent the number orders. How many orders will there be? (What is the value of n as a function of q ?)
- (4p) What is the order size that yields minimal annual cost for the bookstore chain?
- (4p) What is the minimal annual cost?