1331 Final Exam, Fall 2017

You must show your work, and the work you show must yield the answer you obtain, if you are to receive credit. Present the problems in your blue book in the order that they occur on the exam: problem 1 first, problem 2 second, etc. Allow at least one full page for each problem. Note that there is a 20 point bonus question on this exam (#11).

Derivative formulas

$$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx} \qquad \frac{d}{dx}e^u = e^u\frac{du}{dx} \qquad \frac{d}{dx}\ln(u) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx} \qquad \frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Integral formulas

$$\int u^n \frac{du}{dx}dx = \frac{u^{n+1}}{n+1} + C \quad for \quad n \neq 1 \qquad \int \frac{1}{u}\frac{du}{dx}dx = \ln|u| + C$$

$$\int e^u \frac{du}{dx}dx = e^u + C$$

Continuous Income Stream formulas

Present value $= \int_0^k f(t)e^{-rt}dt$ Future value $= e^{rk}\int_0^k f(t)e^{-rt}dt$

1. (20p) **Basic Knowledge and Skills (Derivatives)**. Please compute the following derivatives. (Each part is worth 4 points.)

   a. $\frac{d}{dx}\ 2x^2 - \frac{1}{x^2} + 20$ b. $\frac{d}{dt}t^3e^{2t}$ c. $\frac{d}{dx}\frac{5x+3}{x^2+1}$

   d. $\frac{d}{dp}\frac{1}{\sqrt{3p-1}}$ e. $\frac{d}{dy}\ln(e^{y^2})$

2. (20p) **Basic Knowledge and Skills (Integrals).** Please compute the following integrals. (Each part is worth 4 points.)

   a. $\int \frac{3}{x} - x^2 + 4\,dx$ b. $\int (3x+3)^5\,dx$ c. $\int (2s^2+1)^2\,ds$

   d. $\int ye^{y^2}\ dy$ e. $\int_{-1}^{1} x^2 - 3\,dx$

3. (20p) **Revenue.** A recently released film has its weekly revenue given by

$$R(t) = \frac{50t}{t^2 + 36}.$$

where $R(t)$is in millions of dollars and t is in weeks.

a. (5p) Sketch the graph with the window $0 \leq t \leq 30$ *and* $0 \leq R \leq 10$.
b. (5p) When will the weekly revenue be at its maximum?
c. (5p) What is the maximum weekly revenue?
d. (5p) What is the total revenue for the first 3 weeks?

4. (20p) **Revenue, Cost, Profit.** A company's total revenue function is $R(x) = 1000x$, and its total cost function is $C(x) = 500 + 100x + 0.05x^2$.

a. (5p) What is the total profit function?
b. (5p) Find the marginal revenue function $\overline{MR}(x)$, and the marginal cost function $\overline{MC}(x)$.
c. (5p) Find the production level that will maximize the profit.
d. (5p) What is the maximum profit?

5. (20p) **Revenue.** A table TV company has 4000 customers paying \$110 each month. Suppose each \$1 reduction in price attracts 50 new customers.

a. (5p) Write the equation that gives the monthly revenue $R(x)$ where x is the number of dollars reduction in monthly price.
b. (5p) Find the marginal revenue function.
c. (5p) Find the value of x that maximizes the monthly revenue.
d. (5p) Find the maximum monthly revenue.

6. (20p) **Diminishing Return**s. Suppose the total number of units produced by a worker in t hours of an 8-hour shift can be modeled by the production function $P(t)$:

$$P(t) = 27t + 12t^2 - t^3$$

a. (5p) Sketch the graph of P with window $0 \leq t \leq 8 and\ 0 \leq P \leq 500$.
b. (5p) What is the time at which the total number of units produced will be maximized.
c. (5p) Find the point of diminishing returns. (Where $P'(t)$ is minimized.)
d. (5p) Label the point of diminishing returns on the graph in part a.

7. (20p) **Inventory Model.** Suppose that a company needs 10,000 items during a year and that preparation for each production run costs \$200. Suppose also that it costs \$10 to produce each item and \$1 per year to store an item.

a. (5p) What would be the total cost if the company placed 4 equally spaced orders throughout the year? (Note that each order would be for $\frac{10,000}{4} = 2500$ items.)
b. (5p) What is the number of items in each production run so that the total cost of production and storage is minimized?

c. (5p) What is the number of production runs per year so that the total cost of production and storage is minimized?

d. (5p) What is the minimal annual cost?

8. (20p) **Marginal Cost and Revenue.** A firm's marginal cost for a product is $\overline{MC} = 20x + 20$, and its marginal revenue is $\overline{MR} = 5000$.
   a. (5p) Find the optimal level of production (the value of x that maximizes profit).
   b. (5p) The fixed cost is $1000. Find the total cost function $C(x)$ and the total revenue function $R(x)$.
   c. (5p) Find the Profit function.
   d. (5p) Find the maximum profit.

9. (20p) **Investment.** When the interest on an investment is compounded continuously, the investment grows at a rate that is proportional to the amount in the account, so that if the amount present is P, then

$$\frac{dP}{dt} = kP$$

where P is in dollars, t is in years, and k is a constant.
   a. (5p) Solve this differential equation for the value of P as function of t. (Give the general solution.)
   b. (5p) If $50,000 is invested (when $t = 0$) and the amount in the account after 10 years is $75,000, find the function $P(t)$ that gives the value of the investment after t years.
   c. (5p) What is the interest rate on this investment.
   d. (5p) How many years will it take for the initial investment to double?

10. (20p) **Producer Surplus.** Suppose the demand function for a product is given by $p = 81 - x^2$ and the supply function is given by $p = x^2 + 4x + 11$ where p is the unit price and x is the number of units.
   a. (5p) Sketch the graphs of the demand and supply functions. (Use your calculator with the widow $0 \leq x \leq 10$ *and* $0 \leq p \leq 90$.)
   b. (5p) Compute the equilibrium point and plot it on the graph.
   c. (5p) Identify and shade in the regions that represents the producer's surplus at market equilibrium.
   d. (5p) Compute the producer's surplus at market equilibrium. .

11. (20) **Continuous Income Flow.** The income from an established chain of laundromats is a continuous income stream with annual rate of flow at time t given by $f(t) = 630{,}000$ (dollars per year).
   a. (5p) Sketch the graph of $f(t)$ and shade in the region that represents the total income over the next 5 years.

b. (5p) What will be the total income from this income stream over the next 5 years?
c. (5p) Suppose that the money is worth 7% compounded continuously. What is the present value of the income stream over the next 5 years?
d. (5p) What is the future value of the income stream over the next 5 years?