1331 Final Exam, Spring 2017

You must show your work, and the work you show must yield the answer you obtain, if you are to receive credit. Present the problems in your blue book in the order that they occur on the exam: problem 1 first, problem 2 second, etc. Allow at least one full page for each problem. Note that there is a 20 point bonus question on this exam (#11).

Derivative formulas

$$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx} \qquad \frac{d}{dx}e^u = e^u\frac{du}{dx} \qquad \frac{d}{dx}\ln(u) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}uv = u\frac{dv}{dx} + v\frac{du}{dx} \qquad \frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Integral formulas

$$\int u^n\frac{du}{dx}dx = \frac{u^{n+1}}{n+1} + C \quad for \quad n \neq 1 \qquad \int \frac{1}{u}\frac{du}{dx}dx = \ln|u| + C \qquad \int e^u\frac{du}{dx}dx = e^u + C$$

Continuous Income Stream formulas

Present value $= \int_0^k f(t)e^{-rt}dt$ Future value $= e^{rk}\int_0^k f(t)e^{-rt}dt$

1. (20p) **Basic Knowledge and Skills (Derivatives and Integrals)**. Please compute the following derivatives and integrals (each part is worth 2 points):
   a. $\frac{d}{dx}\frac{1}{\sqrt{x}}$ b. $\frac{d}{dt}3te^{2t}$ c. $\frac{d}{dx}\frac{5x+3}{2x+1}$

   d. $\frac{d}{dp}\sqrt[3]{2p+3}$ e. $\frac{d}{dy}\ln(2y^2+1)$ f. $\int \frac{3}{x} - x\,dx$

   g. $\int (3x+3)^5\,dx$ h. $\int (2s^2+1)^2\,ds$ i. $\int -3e^{-4y}\;dy$

   j. $\int \frac{t^2}{t^3+1}\;dt$

2. (20p). **Pollution.** Consider the function $p = \frac{100C}{7300+C}$.
   a. (4p) What is the natural domain (set of all possible values that C can have) for this function?

b. (4p) Suppose p represents the percent of particulate pollution that can be removed from the smokestacks of an industrial plant by spending C dollars. In this context, what is the domain of the function?

c. (4p) What percentage of the pollution can be removed for \$5,000,000? Round your answer to 3 decimal places.

d. (4p) Find the percent of the pollution that be removed if spending C is allowed to increase without bounds. (As $C \to \infty$.)

e. (4p) Can 100% of the pollution be removed? Explain.

3. (20p) **Marginal Profit.** Suppose the total revenue function is $R(x) = 50x$ and the total cost function is $C(x) = 1900 + 30x + 0.01x^2$.

a. (4p) What is the total profit function?

b. (4p) What is the profit from the sale of 500 units?

c. (4p) Find the marginal profit function $\overline{MP}$.

d. (4p) Find $\overline{MP}$ at $x = 500$.

e. (4p) Find the actual profit from the sale of the 501st item.

4. (20p) **Revenue.** The owner of an apartment building can rent all 50 apartments if she charges \$600 per month, but she rents one fewer apartment for each \$20 increase in monthly rent.

a. (4p) Construct a table that gives the revenue generated if she charges \$600, \$620, \$640.

b. (4p) Write the equation that gives the revenue $R(x)$ from the rental of the apartments if she makes x increases of \$20 in the rent.

c. (4p) Find the marginal revenue function.

d. (4p) Find the value x that maximizes the revenue.

e. (4p) Find the maximum revenue.

5. (20p) **Diminishing Return**s. Suppose a company's daily sales volume attributed to an advertising campaign is given by

$$S(t) = \frac{3}{t+3} - \frac{18}{(t+3)^2} + 1.$$

a. (10p) Compute instantaneous rate of change of $S(t)$.

b. (5p) Find the time at which the sales volume will be maximized.
c. (5p) Find the point of diminishing returns. (Where $S'(t)$ is minimized.)

6. (20p) **Inventory Model.** A restaurant has an annual demand for 1000 bottles of a California wine. It cost \$5 to store 1 bottle for 1 year, it costs \$100 to place an order, and wholesale price to the restaurant is \$15 per bottle. (Assume the demand is constant throughout the year.)
   a. (4p) What is the cost associated with buying, ordering, and storing all 1000 bottles at the beginning of the year?
   b. (4p) What would be the total cost to the restaurant if it placed 2 equally spaced orders throughout the year? (Note that each order would be for $\frac{1000}{2} = 500$ bottles.)
   c. (4p) What would be the total cost function if the restaurant placed n equally spaced orders throughout the year?
   d. (4p) What is the number of orders per year that should be placed in order to minimize the total cost?
   e. (4p) What is the minimal annual cost?

7. (20p) **Minimal Average Cost.** The total daily cost, in dollars, of producing plastic rafts for swimming pools is given by

$$C(x) = 500 + 8x + 0.05x^2$$

   where x is the number of rafts produced per day.
   a. (4p) Find the fixed cost.
   b. (4p) Find the total cost of making 200 rafts.
   c. (4p) Find the average cost function, $\bar{C}(x)$.
   d. (4p) What happens to the average cost as the number of rafts decreases?
   e. (4p) Find the level of production that minimizes average cost.

8. (20p) **Marginal Profit and Profit.** A firm's marginal cost for a product is $\overline{MC} = 2x + 50$, and its marginal revenue is $\overline{MR} = 200 - 4x$, and the cost of production of 10 items is \$700.

a. (4p) Find the optimal level of production. (The value of x that maximizes profit.)
b. (4p) Find the Cost function.
c. (4p) Find the Revenue function.
d. (4p) Find the Profit function.
e. (4p) Find the maximum profit.

9. (20p) **Purchasing Power.** The impact of a 5% inflation rate can be severe. If P represents the purchasing power (in dollars) of an $80,000 pension, then the effect of a 5% inflation rate can be modeled by the differential equation

$$\frac{dP}{dt} = -0.05P$$

where t is in years.

a. (5p) Solve this differential equation for the value of P as function of t. (Give the general solution.)
b. (5p) Find the particular solution for this equation with $P(0) = 80000.$
c. (5p) What happens to the purchasing power as the years go buy.
d. (5p) Find the purchasing power after 15 years.

10. (20p) **Consumer and Producer Surplus.** Suppose the demand function for a product is given by $D(x) = \frac{12}{x+1}$ and the supply function is given by $S(x) = 1 + 0.2x$ where x is the number of units.

a. (4p) Sketch the graphs of $D(x)$ and $S(x)$. Use your calculator with the widow $0 \leq x \leq 20$ and $0 \leq y \leq 15$
b. (4p) Compute the equilibrium point and plot it on the graph
c. (4p) Identify and shade in the regions that represent the consumer's surplus.
d. (4p) Compute the producer's surplus.
e. (4p) Compute the consumers surplus.

11. (20) **Continuous Income Flow.** A company that services a number of vending machines considers its income to be a continuous stream with an annual rate of flow at time t given by $f(t) = 120e^{-0.4t}$ in thousands of dollars per year.

a. (4p) Sketch the graph of $f(t)$. Use your calculator with window $0 \le x \le 10$ and $0 \le y \le 120$.
b. (4p) Shade in the region that represents the total income over the first 5 years.
c. (4p) What will be the total income from this income stream over the first 5 years?
d. (4p) Suppose that the money is worth 10% compounded continuously. What is the present value of the income stream over the next 5 years?
e. (4p) What is the future value of the income stream over the next 5 years?