Math 1331 Final Exam, Spring 2018

You must show all your work - and the work you show must yield the answer you obtain - on each problem in order to receive credit. Present the problems in your blue book in the order that they appear on the exam; if you skip a problem, ensure that you leave enough space to complete it when you return to it later. (A good rule of thumb would be to allot at least one full page for each problem.) Calculators are **NOT** permitted for the exam.

1. (22 pts.) **Basic Knowledge and Skills (Derivatives).** Compute the following derivatives.

   a) (5 pts.) $\frac{d}{dx}\left(\sqrt{x} - 19 + \frac{1}{x^3}\right)$

   b) (5 pts.) $\frac{d}{dy}\ln(4y^2 - 1)$

   c) (6 pts.) $\frac{d}{dt}\left(2te^{3t}\right)$

   d) (6 pts.) $\frac{d}{dx}\left(\frac{x^2+5}{7x-1}\right)$

2. (22 pts.) **Basic Knowledge and Skills (Integrals).** Compute the following integrals.

   a) (5 pts.) $\int \left(4x^2 + \frac{5}{x} - 6\right)\, dx$

   b) (5 pts.) $\int (r^2 - 3)^2\, dr$

   c) (6 pts.) $\int xe^{1-x^2}\, dx$

   d) (6 pts.) $\int \left(\frac{2y^2}{y^3+7}\right)\, dy$

3. (13 pts.) **Rate of Change.** Suppose a waste cleanup company has the ability to remove an amount of waste, in tons, given by $A(C) = \frac{32C}{0.25C+200}$, by spending C dollars. A firm hires them to clean up 120 tons of waste from a demolition site.

   a) (6 pts.) Find the instantaneous rate of change of $A(C)$.

   b) (2 pts.) What is the domain of $A(C)$ in the context of the problem? Explain your answer.

   c) (3 pts.) Find the amount of waste that can be removed if C is allowed to increase without bound ($C \to \infty$).

d) (2 pts.) Can 100% of the waste that the company was hired to clean be removed? Explain your answer.

4. (12 pts.) **Revenue.** A travel agency arranges vacations for large groups of people. For groups of 10 or fewer, they charge \$52 for each person, but for every additional member in the group after 10, everyone in the group receives a \$2 discount. (For example, if the group consists of 12 people, each person is charged \$48.)

   a) (5 pts.) Find the equation for the revenue $R(x)$ that the agency generates for arranging a vacation if x represents the number of people in a group after the tenth (i.e., if there are 11 people in the group, $x = 1$).

   b) (4 pts.) Find the marginal revenue function, $\overline{MR}(x)$.

   c) (3 pts.) Find the **total** number of people in a group that would maximize the agency's revenue.

5. (13 pts.) **Revenue, Cost, Profit.** A company's total cost function for producing x units of a commodity is $C(x) = 0.1x^2 + 11x + 370$ and their revenue function for selling x units is $R(x) = 25x$.

   a) (3 pts.) Find the total profit function.

   b) (3 pts.) Find the marginal revenue function, $\overline{MR}(x)$.

   c) (3 pts.) Find the marginal cost function, $\overline{MC}(x)$.

   d) (4 pts.) Find the production level that maximizes the profit for the company.

6. (18 pts.) **Diminishing Returns.** A company begins an advertising campaign that they determine will result in daily sales volume given by

$$Q(t) = 3 - \frac{10}{(t+2)^2} + \frac{2}{t+2}.$$

   a) (5 pts.) Compute the instantaneous rate of change of $Q(t)$.

   b) (4 pts.) Find the time at which sales volume will be maximized.

   c) (9 pts.) Find the point of diminishing returns for the ad campaign (i.e., the time when $Q'(t)$ is minimized).

7. (16 pts.) **Marginal Cost and Revenue.** A firm's marginal revenue for a product is $\overline{MR}(x) = 90 - 2x$, and its marginal cost is $\overline{MC}(x) = 4x + 24$. The total cost to produce 2 items is \$100.

   a) (3 pts.) Find the optimal level of production (the value of x that maximizes the profit).

   b) (5 pts.) Find the total revenue function $R(x)$.

   c) (5 pts.) Find the total cost function $C(x)$.

   d) (3 pts.) Find the profit function $P(x)$.

8. (12 pts.) **Purchasing Power.** If P represents the purchasing power of a worker's monthly paycheck (in dollars), then the effect of a 1% inflation rate can be modeled by

$$\frac{dP}{dt} = -0.01P$$

   where t is in months.

   a) (5 pts.) Find the general solution for this differential equation for the value of P as a function of t.

   b) (3 pts.) Find the particular solution if a paycheck initially has purchasing power of \$1000 (i.e., if $P(0) = 1000$).

   c) (4 pts.) How long will it be until a paycheck only has purchasing power of \$500? (You may leave your answer in an unsimplified form.)

9. (22 pts.) **Consumer and Producer Surplus.** Suppose that the demand function for a company's product is given by $D(x) = \frac{15}{x+4}$ and the supply function is given by $S(x) = 2 + x$, where each is the unit price in dollars and x is the number of units, in hundreds.

   a) (5 pts.) Compute the equilibrium **price**.

   b) (8 pts.) Compute the producer's surplus at market equilibrium.

   c) (9 pts.) Compute the consumer's surplus at market equilibrium. (You may leave your answer in an unsimplified form.)

10. (+10 pts.) **BONUS: Continuous Income Flow.** A local computer repair shop determines its income to be a continuous stream with an annual rate of flow at time t given by $f(t) = 60e^{0.03t}$, in thousands of dollars per year. Suppose that the money is worth 4% per year compounded continuously.

a) (7 pts.) What is the present value of the income stream over the next 10 years? (You may leave your answer in an unsimplified form.)

b) (3 pts.) What is the future value of the income stream over the next 10 years? (You may leave your answer in an unsimplified form.)