Calculus I: MATH 1351 Final Exam (A)
Fall 2011

Name: _____

Instructions. (Part 1) Solve each of the following problems. Choose the best solution to each problem and clearly mark your choice.

1. Calculate the derivative

$$\frac{d}{dx}\left(x^2 - 2 + \ln x - e^{5x}\right)$$

(a) $2x + -2 + \frac{1}{x} + e^{5x}$
(b) $2x + \frac{1}{x} + 5xe^{4x}$
(c) None of these is correct.
(d) $2x + \frac{1}{x} - 5e^{5x}$
(e) $2x + \frac{1}{x^2} - e^{5x}$

2. Evaluate the integral:

$$\int \sec\theta \tan\theta \, d\theta$$

(a) $\csc\theta + C$
(b) None of these is correct.
(c) $\sec\theta + C$
(d) $\sec^2\theta + C$
(e) $\tan\theta + C$

3. Differentiate $F(x) = \int_5^x (2t - 3)\, dt$.
(a) $F'(x) = 10$
(b) $F'(x) = 0$
(c) $F'(x) = x^2 - 3x - 5$
(d) $F'(x) = 2x - 3$
(e) None of these is correct.

4. Find the limit

$$\lim_{x\to 0} \frac{|x|}{x}$$

(a) 0
(b) -1
(c) Limit does not exist.
(d) -2
(e) 1

5. Use right-endpoint sum approximation to estimate the area under the graph of the function $f(x) = x^2$ between 0 and 1 using a partition step size of $\frac{1}{2}$ (i.e., using two rectangles of equal width).

(a) 1/4

(b) 3/4

(c) 1/3

(d) None of these is correct.

(e) 5/8

6. For the function $f(x) = x^2 + 2x + 2$, find all numbers c between -2 and 1 that satisfy the conclusion of the Mean Value Theorem, i.e., find all numbers c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

(a) There is no such number c; the Mean Value Theorem does not apply.

(b) $c = 0, 1/2$

(c) $c = -2, 1$

(d) $c = -1/2, 1/2$

(e) $c = -1/2$

7. Given $y = \sin^{-1}(\cos x)$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$.

(a) $\frac{dy}{dx} = -1$

(b) $\frac{dy}{dx} = \frac{-1}{\sqrt{2}}$

(c) $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$

(d) $\frac{dy}{dx} = \frac{-1}{\sqrt{1-\frac{\pi^2}{4}}}$

(e) None of these is correct.

8. Evaluate the integral:

$$\int_{-1}^{1} \sqrt{x+1}\, dx$$

(a) $-\frac{2}{3}\sqrt{2}$

(b) $\frac{4}{3}\sqrt{2}$

(c) $\frac{1}{2\sqrt{2}}$

(d) None of these is correct.

(e) $-\frac{4}{3}\sqrt{2}$

9. Evaluate the integral:

$$\int \frac{e^x\,dx}{1+e^{2x}}$$

(a) None of these is correct.

(b) $e^x \tan^{-1}(e^x) + C$

(c) $\ln(1+e^{2x}) + C$

(d) $\tan^{-1}(e^x) + C$

(e) $-2(1+e^{2x})^{-2} + C$

10. If $y = x^3 + 6x^2 - 5x + 10$, what is $\frac{d^4y}{dx^4}$?

(a) 6

(b) x

(c) The 4th derivative does not exist.

(d) 0

(e) x^3

11. Find the absolute extreme values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on the interval $[-2, 2]$.

(a) Absolute maximum is 20 at $x = -2$; absolute minimum is -7 at $x = 1$.

(b) Absolute maximum is 4 at $x = 2$; absolute minimum is 0 at $x = 0$.

(c) Absolute maximum is 20 at $x = -2$; absolute minimum is 4 at $x = 2$.

(d) None of these is correct.

(e) Absolute maximum is 0 at $x = 0$; absolute minimum is -7 at $x = 1$.

12. Evaluate the limit:

$$\lim_{x\to 1} \frac{x}{x^2-1}$$

(a) 0

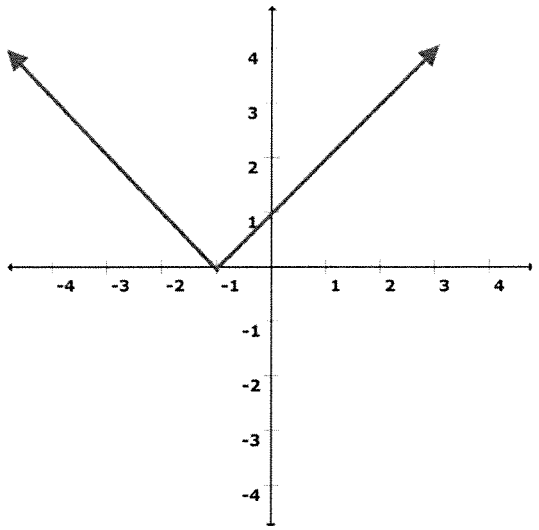
(b) 2

(c) Limit does not exist.

(d) -1/2

(e) 1/2

13. The following figure shows the graph of a function $f(x)$. At the point $x = -1$, does the graph appear to be differentiable, continuous but not differentiable?



(a) None of these is correct.

(b) Neither Continuous nor differentiable.

(c) Continuous and differentiable.

(d) Continuous, but not differentiable.

(e) Differentiable, but not continuous.

14. Evaluate the following limit:

$$\lim_{x \to -\infty} xe^x$$

(a) ∞/∞

(b) 0

(c) None of these is correct.

(d) -1

(e) ∞

15. Which of the following statements are true?

I. If $x = c$ is a critical number of a continuous function f, then $f(c)$ must be either a relative maximum or a relative minimum.

II. If $x = c$ is a critical number of a continuous function f, then c is in the domain of f and either $f'(c) = 0$ or $f'(c)$ does not exist.

III. If a continuous function f has a relative maximum or minimum at $x = c$, then c must be a critical number of f.

(a) II and III.

(b) I and III.

(c) I, II, and III.

(d) II only.

(e) I only.

16. Let $s(t) = t^2 - 2t + 6$ for $0 \leq t \leq 2$ denote the position of an object moving along the line. Find the acceleration at time t; find where the acceleration is positive.

(a) $a(t) = 2$; acceleration positive on $[0, 2]$

(b) $a(t) = 1$; acceleration positive on $[0, 1]$

(c) None of these is correct.

(d) $a(t) = 2t - 2$; acceleration positive on $[1, 2]$

(e) $a(t) = 2t - 2$; acceleration positive on $[0, 2]$

17. If $\int_1^3 f(x)\,dx = 6$ then $\int_3^1 f(x)\,dx =$

(a) None of these is correct.

(b) $-1/6$

(c) 6

(d) -6

(e) $1/6$

18. Find the average value of the function $y = x^2$ over the interval $[0, 2]$.

(a) 2

(b) $8/3$

(c) $4/3$

(d) None of these is correct.

(e) 4

19. Evaluate the indefinite integral:

$$\int \left(\frac{\sqrt{y}}{2} + \frac{1}{\sqrt{y}} \right) dy$$

(a) $\frac{1}{3}y^{3/2} + 2\sqrt{y} + C$

(b) $\frac{1}{3}y^{3/2} + \ln(\sqrt{y}) + C$

(c) None of these is correct.

(d) $\frac{3}{2}y^{3/2} + \frac{1}{2}\sqrt{y} + C$

(e) $\frac{1}{3}\sqrt{y} - \frac{1}{2\sqrt{y}} + C$

20. Evaluate the following limit:

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{\cos x}{x-\frac{\pi}{2}}\right)$$

(a) -1

(b) 1

(c) $\frac{0}{0}$

(d) Limit does not exist.

(e) None of these is correct.

21. Find a constant number a so that $f(x)$ is continuous at every point:

$$f(x)=\begin{cases} x^3, & x\leq 2 \\ ax^2-4, & x>2 \end{cases}$$

(a) $a=-2$

(b) $a=8$

(c) $a=2$

(d) $a=3$

(e) None of these is correct.

22. Find $g'(t)$ where $g(t)=te^{1-2t}$.

(a) $g'(t)=(1+t)e^{1-2t}$

(b) $g'(t)=te^{1-2t}$

(c) $g'(t)=(1-2t)^2e^{1-2t}$

(d) None of these is correct.

(e) $g'(t)=(1-2t)e^{1-2t}$

23. Simplify the expression $7^{\log_7(4x)}$.

(a) 1

(b) 7^{4x}

(c) 7

(d) None of these is correct.

(e) $4x$

24. Find the exact value of $\sec\left(\frac{\pi}{4}\right)$.

(a) $\frac{\sqrt{2}}{2}$

(b) $\sqrt{2}$

(c) $\frac{2\sqrt{3}}{3}$

(d) $\frac{1}{2}$

(e) None of these is correct.

Instructions. (Part 2) Solve the following problem. Show your work clearly and write out all relevant steps.

25. Use the definition of the derivative (as a limit of the difference quotient) to find $f'(x)$ for $f(x) = 4 - 5x$.

26. Find the equation of the tangent line to the curve $x^2 + 2xy = y^3$ at the point $(-1, 1)$.

27. If pressure at time t is related to volume at time t by $PV = 100$, and if the volume is changing at a rate of 10 cubic inches per second, then at what rate is the pressure changing when $V = 10$?

28. A closed box with a square base is to be built to house an ant colony. The bottom of the box and all four sides are to be made of material costing $\$1/ft^2$, and the top is to be constructed of glass costing $\$5/ft^2$. What are the dimensions of the box of greatest volume that can be constructed for \$72?

29. Given the following function and its derivatives:

$$f(x) = \frac{3x+5}{7-x}, \qquad f'(x) = \frac{26}{(x-7)^2}, \qquad f''(x) = \frac{-52}{(x-7)^3}$$

a. Find any vertical or horizontal asymptotes.

b. Find all intervals on which the function is increasing and all intervals on which it is decreasing.

c. Find any critical points or points of inflection.

d. Find all intervals on which the function is concave up on all intervals on which it is concave down.

e. Find any relative maxima or relative minima.

f. Sketch a graph of the function that accurately depicts the features from parts (a) through (e). Clearly label the features from parts (a) through (e) on your graph.