Math 1351 Final Exam - Spring 2011
Show all relevant work. No calculators or electronics of any kind.

1. Solve the following equation for x:

$$\frac{e^{x^2}}{e^{x+6}} = 1$$

2. Using the limit definition of the derivative, find the derivative of $f(x) = \cos x$.

3. Find an equation for the tangent line to the curve $y = x\cos x$ at $x = \frac{\pi}{2}$.

4. Let $f(x) = 4x^3 - 12x^2 + 10$. Find and identify each of the following (if they exist) and sketch a graph of $f(x)$ clearly indicating values from (a) through (e):

   (a) All critical numbers for $f(x)$.

   (b) All intervals on which the graph of $f(x)$ is increasing; all intervals on which the graph is decreasing.

   (c) Any x values for which $f(x)$ achieves a relative maximum; any x values for which $f(x)$ achieves a relative minimum.

   (d) All intervals on which the graph of $f(x)$ is concave up; all intervals on which the graph is concave down.

   (e) The x values of any inflection points of the graph of $f(x)$.

5. Find the absolute maximum and minimum values of the function $f(x) = x^2 - 4x - 3$ on the interval $[-2, 4]$.

6. For the function $g(x) = \frac{2x}{x+3}$, find all vertical and horizontal asymptotes of the graph.

7. Find $\frac{dy}{dx}$:

   (a) $y = \sqrt{4x} - x^2 + \ln(x^2+1)$

   (b) $y = \tan^{-1}(2x+5)$

   (c) $y = \frac{e^x}{x - \csc x}$

8. Use implicit differentiation to find $\frac{dy}{dx}$ if $xy + y^3 = 10$.

9. When a circular plate of metal is heated in an oven, its radius increases at a rate of 0.01 cm/min. At what rate is the plate's area increasing at the moment when the radius is 25 cm?

10. Evaluate the following limits:

    (a) $\displaystyle\lim_{x\to 0} \frac{\sin^2 x}{2x}$

    (b) $\displaystyle\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

    (c) $\displaystyle\lim_{x\to 0} (e^x + x)^{1/x}$

11. A jogger is 25 miles due east of a skate boarder and is traveling west at a constant speed of 6 miles per hour. Meanwhile, the skate boarder is going north at 8 miles per hour. When will the jogger and the skate boarder be closest to each other? What is the minimum distance between them? (Hint: let S equal the square of the distance and minimize S.)

12. Let $f(x) = 2x + 1$ on $[0, 1]$:

    (a) Set up the Riemann sum $\sum_{k=1}^{n} f(a+k\Delta x)\Delta x$ for $f(x)$ on $[0, 1]$ using equal-width rectangles and evaluating the function at right endpoints.

    (b) Estimate the area bounded by the graph of $f(x)$, the x-axis, and the lines $x = 0$ and $x = 1$ using the Riemann sum from (a) with four rectangles ($n = 4$) .

    (c) Find the area bounded by the graph of $f(x)$, the x-axis, and the lines $x = 0$ and $x = 1$ by evaluating the sum in (a) and taking the limit as $n \to \infty$.

    (d) Check your answer from (c) by evaluating a definite integral.

13. Evaluate the following integrals:

    (a) $\displaystyle\int \frac{x^2 - 3x + \sqrt{x}}{x^2}\,dx$

    (b) $\displaystyle\int \frac{3-x}{\sqrt{1-x^2}}\,dx$

    (c) $\displaystyle\int \sin 2\theta \, d\theta$

14. Find the area of the region under the curve $y = \sec^2 x$ on $[0, \frac{\pi}{4}]$.

15. Find $G'(x)$, where $G(x) = \int_x^1 \frac{dt}{\sqrt{1+3t^2}}$

16. Evaluate the following definite integrals:

    (a) $\displaystyle\int_0^4 x\sqrt{x^2+9}\,dx$

    (b) $\displaystyle\int_{e^{-1}}^{e} \frac{dx}{x}$

    (c) $\displaystyle\int_0^1 \frac{4}{1+x^2}\,dx$