MATH 1451 Final
Fall 2012

1. Decide whether the function $f(x) = \frac{x^2-1}{x-1}$ is continuous everywhere. If not, define or redefine the function at one point to make it continuous everywhere.

2. Show that the equation $x^3 + 3 = 4x$ has at least one solution on the interval $(-3, -2)$.

3. Solve for x : $\ln(x+1) + \ln\left[\frac{1}{(x+1)(x^2-1)}\right] = \ln\frac{1}{3}$

4. Find the second derivative of $f(x) = x^2 \sin(x)$.

5. A person standing on the edge of a building 320 feet tall throws a ball straight up into the air with an initial velocity of 128 feet per second.
   (a) Write down the formula for the velocity of the ball at time t.
   (b) At what time t will the ball hit the ground?
   (c) What is the velocity of the ball at the instant the ball hits the ground?

6. Differentiate the following function: $f(x) = \ln\sqrt{x^2+2x+4}$

7. Use implicit differentiation to find $\frac{dy}{dx}$, $\quad xy^2 + \sin y = x^3$.

8. Evaluate $\quad \lim_{x\to 2}\frac{x^2+x+4}{x^2+3x-4}$

9. Consider the piecewise function $f(x) = \begin{cases} 4-x^2, & if\ x \le 2 \\ x-1, & if\ x > 2 \end{cases}$

   (a) Find $f(2)$

   (b) Find $\lim_{x\to 2^-} f(x)$

   (c) Find $\lim_{x\to 2^+} f(x)$

   (d) Find $\lim_{x\to 2} f(x)$

   (e) Is f continuous at $x = 2$? (Justify answer)

10. Evaluate the following limit,

$$\lim_{x\to\infty} \frac{3x^4 + 2x + 5}{5x^{4.5} + 8x^3}.$$

11. Find $\lim_{x\to\pi/2} \frac{3\sec x}{2+\tan x}$.

12. Find $\lim_{x\to 0^+} (e^x + x)^{1/x}$.

13. What is length of the base of the rectangle with largest area that can be inscribed in a semicircle of radius 10, assuming that the base of the rectangle lies on the diameter of the semicircle?

14. The highway department is planning to build a rectangular picnic area for motorists along a highway. It is to have an area of 7,200 square yards and is to be fenced off on the three sides not adjacent to the highway. What is the least amount of fencing that will be needed to complete the job?

15. A teacher is trying to make a right rectangular box that is open at the top and that is as large as possible out of a piece of 5 in by 8 in piece of cardboard by cutting squares out of the corners and then folding up the sides. What is the height of the largest such box that can be made?

16. Evaluate the following integrals:

(a) $\int_0^{\frac{\pi}{4}} (\sec^2 x - \sin x)dx$

(b) $\int_1^2 \left(\frac{1}{x} + x\right) dx$

17. Evaluate the following integrals:

(a) $\int_1^3 \left(x^2 + \sqrt{x} + \frac{1}{x^2}\right) dx$

(b) $\int_0^{\frac{1}{2}} \frac{4}{\sqrt{1-x^2}} dx$

18. Find $\frac{dy}{dx}$ when $y(x)$ is defined as

$$y(x) = \int_x^1 \frac{dt}{\sqrt{7t^2 + 1}}.$$

19. Find the indefinite integral $\int \sin^3\theta \cos\theta \, d\theta$.

20. Evaluate the integral $\int_1^2 \sqrt{x}\, e^{x\sqrt{x}} dx$.