Instructions.   Solve each of the following problems.   Choose the best solution to each problem and clearly mark your choice.

1. Find the absolute extrema of $f(x) = 2x^3 - 3x^2$ on the interval $[-1, 2]$.

   (a) Absolute minimum is -5 and absolute maximum is 4.

   (b) Absolute minimum is -1 and absolute maximum is 0.

   (c) Absolute minimum is -5 and absolute maximum is 0.

   (d) Absolute minimum is -6 and absolute maximum is 5.

   (e) Absolute minimum is 0 and absolute maximum is 4.

2. Evaluate the following limit: $\displaystyle\lim_{x \to 1^-} \frac{1}{x-1}$.

   (a) 1

   (b) $-\infty$

   (c) -1

   (d) ∞

   (e) 0

3. Evaluate the following limit: $\displaystyle\lim_{x \to 2} \frac{\sin(\pi x)}{x-2}$.

   (a) 0

   (b) ∞

   (c) 1

   (d) $\frac{\pi}{2}$

   (e) π

4. What is the domain of $f(x) = \sin^{-1}(x)$?

   (a) $[-\pi, \pi]$

   (b) $[0, \pi]$

   (c) $[0, 1]$

   (d) $[-1, 1]$

   (e) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

5. If $y = \csc^3(x^2)$, then what is $\frac{dy}{dx}$?

   (a) $-6x \csc^3(x^2) \cot(x^2)$

   (b) $\csc(x^6) \cdot \cot(x^6)$

   (c) $6x \csc^3(x^2) \cot(x^2)$

   (d) $-6x \csc^2(x^2) \cot(x^2)$

   (e) $6x \csc^4(x^2)$

6. Find $\displaystyle\int_1^2 \frac{x^3 - 1}{x^2}\,dx.$

   (a) Undefined because we are integrating over an asymptote

   (b) $\dfrac{1}{4}$

   (c) 2

   (d) 3

   (e) 1

7. If $y = e^{2x}\cos(3x)$, then what is $\frac{dy}{dx}$?

   (a) $e^{2x}\cos(3x) + 3e^{2x}\sin(3x)$

   (b) $\dfrac{e^{2x+1}\sin(3x)}{6}$

   (c) $-6e^{2x}\sin(3x)$

   (d) $2e^{2x}\cos(3x) - 3e^{2x}\sin(3x)$

   (e) $e^{2x}\cos(3x) - e^{2x}\sin(3x)$

8. Find all of the horizontal and vertical asymptotes of $2 + \dfrac{x^2 - 4}{x(x-2)}$?

   (a) $y = 3$ and $x = 0$

   (b) $y = 1$ and $x = 0$

   (c) $y = 3$ and $x = 0$, $x = 2$

   (d) $y = 1$, $y = 2$ and $x = 0$, $x = 2$

   (e) $y = 2$ and $x = 2$

9. If $y = \tan^{-1}\left(\dfrac{x}{3}\right)$, then what is $\frac{dy}{dx}$?

   (a) $\dfrac{1}{1 + x^2}$

   (b) $\dfrac{3}{9 + x^2}$

   (c) $\dfrac{1}{3}\dfrac{1}{1 + x^2}$

   (d) $\dfrac{3}{|x|\sqrt{x^2 - 1}}$

   (e) $\dfrac{1}{\sqrt{9 - x^2}}$

10. Find $\displaystyle\int_0^2 x\sqrt{x^2+1}\,dx$.

    (a) Undefined because we are integrating over an asymptote.

    (b) $\dfrac{1-5\sqrt{5}}{3}$

    (c) $\dfrac{5\sqrt{5}-1}{3}$

    (d) $\dfrac{1}{3}$

    (e) $\dfrac{1}{6}$

11. Find the constants a and b that would make the following function continuous:

$$f(x) = \begin{cases} x^2 - 4x + b + 3 & \text{if } x \text{ less than } 1; \\ 3 & \text{if } x = 1; \\ ax + b & \text{if } 1 \text{ less than } x. \end{cases} \tag{1}$$

    (a) $a = 0, b = 3$

    (b) $a = 1, b = 2$

    (c) $a = -1, b = 4$

    (d) $a = 3, b = 0$

    (e) $a = 0, b = -3$

12. Evaluate the following limit: $\displaystyle\lim_{x\to 0} \frac{\sin\left(x + \frac{\pi}{2}\right)}{x + \frac{\pi}{2}}$.

    (a) 0

    (b) ∞

    (c) $\frac{\pi}{2}$

    (d) $\frac{2}{\pi}$

    (e) 1

13. A truck is 250 mi due east of a sports car and is traveling west at a constant speed of 60 mi/h. Meanwhile, the sports car is going north at 80 mi/h. When will the truck and the car be closest to each other?

    (a) In 2 hours.

    (b) In 200 hours.

    (c) In $\frac{3}{4}$ of an hour.

    (d) In 1 and $\frac{1}{2}$ hours.

    (e) In 40 minutes.

14. Find the average value of $f(x) = 4x^3$ on $[1, 2]$.

(a) 36

(b) $\frac{15}{2}$

(c) $\frac{27}{2}$

(d) 15

(e) $\frac{3}{2}$

15. Find $\displaystyle\int x^3 - x^{-2} + x^{-1}dx$.

(a) $\dfrac{x^4}{4} + x^{-1} + \ln|x| + C$

(b) $\dfrac{x^4}{4} - x^{-1} + \ln|x| + C$

(c) $\dfrac{x^4}{4} + x^{-1} - x^{-2} + C$

(d) $\dfrac{x^2}{2} - 2x^{-3} + \ln|x| + C$

(e) $\dfrac{x^4}{4} - 2x^{-3} - \ln|x| + C$

16. The volume of a spherical balloon is increasing at a constant rate of 3 cubic inches per second. At what rate is the radius of the balloon increasing when the radius is 2 in.?

(a) $\dfrac{3}{32\pi}$ in/s

(b) $\dfrac{3}{2\pi}$ in/s

(c) $\dfrac{5}{32\pi}$ in/s

(d) $\dfrac{5}{16\pi}$ in/s

(e) $\dfrac{3}{16\pi}$ in/s

17. What is the formal definition of $f'(x)$?

(a) $\displaystyle\lim_{\Delta x \to 0} \dfrac{f'(x + \Delta x) - f'(x)}{\Delta x}$

(b) $\displaystyle\lim_{h \to 0} \dfrac{f(x + h) - f(x)}{h - x}$

(c) $\displaystyle\lim_{\Delta x \to 0} \dfrac{f(x) - f(x + \Delta x)}{\Delta x}$

(d) $\displaystyle\lim_{\Delta x \to 0} \dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

(e) $\displaystyle\lim_{\Delta x \to 0} \dfrac{f(x + \Delta x) + f(x)}{\Delta x}$

18. Find $\int \dfrac{2}{\sqrt{1-(2x)^2}}dx$.

(a) $\sin(2x)+C$

(b) $\sec^{-1}(2x)+C$

(c) $\dfrac{1}{\sin^{-1}(2x)}+C$

(d) $\sin^{-1}(2x)+C$

(e) $F(x)=\displaystyle\int_1^x \sec^{-1}(2x)dx$

19. If $y=\dfrac{7}{x^2}+x^{\frac{2}{3}}$, then what is $\dfrac{dy}{dx}$?

(a) $7\ln|x|+\dfrac{2}{3\sqrt[3]{x}}$

(b) $-14x^{-3}+\dfrac{2}{3}x^{-\frac{1}{3}}$

(c) $-7x^{-1}+\dfrac{2}{3}x^{\frac{1}{3}}$

(d) $\dfrac{7}{2x}+\dfrac{2}{3}x^{\frac{2}{3}}$

(e) $14x+\dfrac{2}{3}x$

20. Evaluate the following limit: $\displaystyle\lim_{h\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$.

(a) Undefined

(b) $\frac{1}{2}x^{-\frac{1}{2}}$

(c) $\sqrt{x}$

(d) 1

(e) $\frac{\pi}{2}$

21. If $xy^2=x+y$, then what is $\dfrac{dy}{dx}$?

(a) $x(y^2-1)$

(b) $\dfrac{1-y^2}{2xy-1}$

(c) $\dfrac{1-y}{2y^2-1}$

(d) $\dfrac{1-y}{2x-1}$

(e) $\dfrac{1}{2y-1}$

22. Assume a person standing on top of a 128 foot building throws a ball directly upward with the initial speed of 32 ft/s. When the ball hits the ground what is its impact velocity?

(a) -128 ft/s

(b) -96 ft/s

(c) -64 ft/s

(d) -32 ft/s^2

(e) 32 ft/s

23. Let $f(x) = 3x^5 - 5x^3 + 2$. What are the intervals where $f(x)$ is increasing?

(a) $(-\infty, 0) \cup (0, 1)$

(b) $(-1, 1)$

(c) $(-\infty, -1) \cup (1, \infty)$

(d) $-(\frac{\sqrt{2}}{2}, 0)$

(e) $(-1, 0) \cup (1, \infty)$

24. If $y = \ln(\sec(x))$, then what is $\frac{dy}{dx}$?

(a) $\cot(x)$

(b) $\sec(x)$

(c) $-\ln|\cos(x)|$

(d) $\tan(x)$

(e) $\sec^{-1}(x)$

25. If $y = \dfrac{4}{\sqrt{x}}$, then what is $\frac{d^2y}{dx^2}$?

(a) $2\sqrt{x}$

(b) $3x^{\frac{5}{2}}$

(c) $\dfrac{3}{x^2\sqrt{x}}$

(d) $\dfrac{6}{x^2\sqrt{x}}$

(e) $-x^{-\frac{3}{2}}$