FINAL EXAM 1451 A

(FALL 2013)

Please turn off and put away your cell phones. **Use of calculators is not allowed.** To receive full credit show all your work and circle your final answer.

(1) Evaluate the following limits:

(i) $\lim_{x\to 2} \dfrac{x^4-16}{x-2}$

(ii) $\lim_{x\to\infty} \left(\dfrac{e^{-x}+1}{2} + \dfrac{2x^2-x}{4x^2-1}\right)$

(iii) $\lim_{x\to 2} \dfrac{\cos(\frac{\pi}{x})}{x^2-4}$

(2) For each function below find $\frac{dy}{dx}$:

(i) $y = x^3 - \dfrac{1}{x} + \sqrt{x}$

(ii) $y = \dfrac{\sin(x)}{x}$

(iii) $y = \sin(x^2)\cos(x^3)$

(iv) $y = \ln(\sec(x^2))$

(3) Using implicit differentiation find y' when $xy = x^2 + y^2$.

(4) Find the equation for the line through the point $(\pi, \frac{\sqrt{2}}{2})$ tangent to $f(x) = \cos(x/4)$.

(5) A farmer has 256 feet of fence and wishes to enclose a rectangular plot of land. The land has a barn on one edge and no fence is required on that side. What should the length of the side parallel to the barn be in order to include the largest possible area?

(6) A spherical balloon is being filled with a gas in such a way that when the radius is 2 ft, the radius is increasing at the rate of 1/8 ft/min. How fast is the volume changing at this time?

(7) Given the function $f(x) = 1 + \frac{3(x-1)(x+2)}{x^2-x}$ find the horizontal and vertical asymptotes.

(8) Use the function $f(x) = -x^4 + 2x^3 + 3$ to answer the questions below:

(i) Where does f have relative maximums?

(ii) Where does f have relative minimums?

(iii) At what intervals is f concave up?

(iv) At what intervals is f concave down?

(v) Where does f have inflection points?

(9) Find the indefinite integrals below:

(i) $\int (2x^3 - x^{-1} + x^{-2})dx$

(ii) $\int \sin(x)(\cos(x))^3 dx$

(10) Evaluate the following definite integrals:

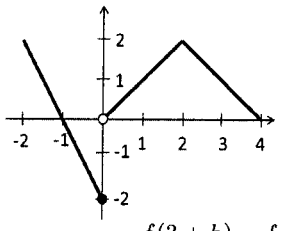
(i) $\int_0^{\pi} \cos\left(\dfrac{\theta}{2}\right) d\theta$

(ii) $\int_0^1 2x(x^2+1)^3 dx$

(11) Approximate $\int_1^5 x^2 dx$ using the Trapezoid Rule with $n = 4$, (using 4 trapezoids).

(12) For the subsequent questions use the following definition of f:

$$f(x) = \begin{cases} -2(x+1) & \text{if } x \le 0; \\ 2 - |x-2| & \text{if } 0 < x. \end{cases}$$



(i) What is $\lim_{x\to 0} f(x)$?

(ii) What is $\lim_{h\to 0} \dfrac{f(3+h)-f(3)}{h}$?

(iii) What is $\int_1^4 f(x)dx$?