FINAL EXAM 1451

Please turn off and put away your cell phones. **Use of calculators is not allowed.** To receive full credit show all your work and circle your final answer.

(1) Evaluate the following limits:
   (a) $\lim_{x\to 1} \frac{x}{x+1}$
   (b) $\lim_{x\to\infty} \left(\frac{\cos(2x)}{x} + 2\right)$
   (c) $\lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6n^3}$
   (d) $\lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$
   (e) $\lim_{x\to\infty} \frac{\ln(x)}{x^2}$
   (f) $\lim_{x\to 1} \frac{x^3 - 1}{x - 1}$
   (g) $\lim_{x\to\infty} \left(1 + \frac{1}{2x}\right)^{2x}$

(2) Using the definition of derivative find $\frac{df}{dx}$ at $x = 1$ where $f(x) = x^2 + 3x$.

(3) For each function below find $\frac{dy}{dx}$:
   (a) $y = 5x^4 - 3x + x^{-2} - \frac{1}{x^3}$
   (b) $y = \frac{x^2}{\cos(x)}$
   (c) $y = \sin^{-1}(3x)$
   (d) $y = 3^x - \ln(\cos(x))$
   (e) $y = \csc(e^{x^2})$
   (f) $y = x^3 \sin(x)$

(4) Using implicit differentiation find y' when $y^2\cos(x) = x$.

(5) Find the equation for the line through the point $(1, 2)$ tangent to $f(x) = x^3 + 1$.

(6) A ladder 13 ft long rests against a vertical wall and is sliding down the wall at the rate of 3 ft/s at the instant the foot of the ladder is 5 ft from the base of the wall. At this instant, how fast is the foot of the ladder moving away from the wall?

(7) A poster is to contain 108 cm^2 of printed matter, with margins of 6 cm each at top and bottom and 2 cm on the sides. What is the minimum cost of the poster if it is made of material costing 10 cents per cm^2?

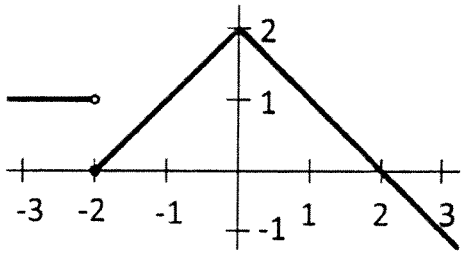
(8) Given the function $f(x) = \frac{3x(x-1)}{x(x-2)}$ find the horizontal and vertical asymptotes.

(9) Find the area bounded by the x-axis and the graph of $f(x) = 1 - x^2$.

(10) Use the function $f(x) = x^4 - 4x^3 + 10$ to answer the questions below:
   (a) Where does f have relative maximums?
   (b) Where does f have relative minimums?
   (c) At what intervals is f concave up?
   (d) At what intervals is f concave down?
   (e) Where does f have inflection points?

(11) Using this definition of f answer the following questions.

$$f(x) = \begin{cases} 1 & \text{if } x < -2; \\ 2 - |x| & \text{if } -2 \le x. \end{cases}$$



   (a) What is $\lim_{x\to -2^-} f(x)$?
   (b) What is $\lim_{x\to -2^+} f(x)$?
   (c) What is $\lim_{x\to -2} f(x)$?
   (d) What is $f'(-1)$?
   (e) What is $\lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$?
   (f) What is $\lim_{n\to\infty} \left(\sum_{i=1}^{n} \left[f\left(0 + i\frac{2}{n}\right) \cdot \frac{2}{n}\right]\right)$?
   (g) What are the values of the domain where f is not continuous?
   (h) What are the values of the domain where f does not have a derivative?
   (i) What is $\int_{-1}^{3} f(x)dx$?

(12) Find the indefinite integrals below:
   (a) $\int (8x^3 + x^{-1} + x^{\frac{-1}{2}})dx$
   (b) $\int \frac{4}{1 + 4x^2} dx$
   (c) $\int (4x + 2)(x^2 + x - 1)^5 dx$

(13) Evaluate the following definite integrals:
   (a) $\int_0^{\pi} \frac{1 + \cos(2\theta)}{2} d\theta$
   (b) $\int_{-1}^{1} 1dx$
   (c) $\int_0^2 xe^{x^2} dx$