

**Final Exam 1451 (Fall 2014)**

To receive full credit show all your work and circle your final answer.

(1) Evaluate the following limits:

(i)  $\lim_{x \rightarrow 3^-} \frac{1}{(3-x)^2}$

(ii)  $\lim_{x \rightarrow \infty} \frac{4x^3 - 4x^2 + 1}{-2x^3 + 10}$

(iii)  $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^{2x}$

(iv)  $\lim_{x \rightarrow -\infty} \left(\frac{4(3^x)}{3^x - 9}\right)$

(2) For each function below find  $\frac{dy}{dx}$ :

(i)  $y = x^{-1} + x^{-2} + 2^x$

(ii)  $y = \tan^{-1}(3x)$

(iii)  $y = \sec(2x) \tan(2x)$

(iv)  $y = \frac{e^x}{x}$

(3) Using implicit differentiation find  $y'$  when  $2xy = x^3 + y^3$ .

(4) Find the equation for the line through the point  $(1, 1)$  tangent to curve defined in preceding problem.

(5) A rancher wishes to enclose a rectangular plot of land with 1200 feet of fence. The land has a river on one side and no fence is required on that side. What should the length of the side parallel to the river be in order to enclose the largest possible area?

(6) Sand is being emptied from a conveyor belt at the rate of 15 ft<sup>3</sup>/s. The sand forms a conical pile whose height is always twice its radius. At what rate is the radius of the pile increasing when its radius is 5ft? (Recall that volume of a cone is given by  $V = \frac{1}{3}h\pi r^2$ )

(7) Given the function  $f(x) = \frac{x^2-1}{x^2+3x+2}$  find the horizontal and vertical asymptotes.

(8) Use the function  $f(x) = x^3 + 3x^2 - 9x + 2$  to answer the questions below:

- (i) Where does  $f$  have relative maximums?      (ii) Where does  $f$  have relative minimums?

(9) Use the function  $f(x) = 3x^5 - 5x^4 + 5x + 1$  to answer the questions below:

- (i) Where is  $f$  concave down?      (ii) Where is  $f$  concave up?

(iii) Where does  $f$  have inflection points?

(10) Find the indefinite integrals below:

(i)  $\int (8x^3 - 2x^{-1} + \sqrt{x}) dx$

(ii)  $\int \frac{1}{x \ln(x)} dx$

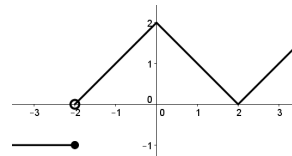
(11) Evaluate the following definite integrals:

(i)  $\int_0^{\frac{\pi}{2}} \sin(\theta) d\theta$

(ii)  $\int_{-1}^0 (2x+1)(2x^2+2x-1)^3 dx$

(12) For the subsequent questions use the following definition of  $f$ . All questions can be answered by examining the graph of the function.

$$f(x) = \begin{cases} -1 & \text{if } x \leq -2; \\ 2 - |x| & \text{if } -2 < x < 2. \\ x - 2 & \text{if } 2 \leq x. \end{cases}$$



(i) What is  $\lim_{x \rightarrow -2^+} f(x)$ ?

(ii) What is  $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$ ?

(iii) What is  $\lim_{n \rightarrow \infty} \left( \sum_{i=1}^n \left[ f\left(-1 + i \frac{2}{n}\right) \cdot \frac{2}{n} \right] \right)$ ?