Math 1451 Final Exam
Spring 2014

1. Evaluate $\lim_{x\to 0}\frac{\sin^2 x}{x^2}$.

2. Find the constant a such that the function f be continuous for all x
$$f(x)=\begin{cases} a\sin(x)\,; & if\ 0\le x<\frac{\pi}{2} \\ \pi-x\quad ; & if\ \frac{\pi}{2}\le x<\pi \end{cases}$$

3. Solve for x in
$\frac{e^{x^2}}{e^{-5x+14}}=1$.

4. Let $f(x)=(1-x)^{2/3}+\frac{1}{3}x$. Find all critical points of the function f.

5. Let $g(x)=\tan^{-1}(2x)$. Find the absolute maximum and minimum values on the interval $\left[-\frac{1}{2},\frac{1}{2}\right]$.

6. Using the formal definition of derivative $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
prove that $\frac{d(u^3)}{du}=3u^2$.

7. Find the derivative of the function k, where it exists:
$k(u)=\frac{u^2+\pi^2}{u-\pi}+2\sqrt{u}+\frac{1}{u^5}+2014\pi$.

8. Find the derivative of the following function, where it exists:
$f(x)=\ln\left(\cos\left(\sqrt[3]{x+2}\right)\right)$.

9. A spherical ball is to be deflated so that its radius decreases at a constant rate of 8cm/min. Calculate the rate of change of the volume at the instant when the radius is 5 cm.

10. If $h(x)=\frac{x\sin x}{e^x}$, find $h'(\pi/4)$.

11. Find the x-coordinate of each point on the graph of $f(x)=x^2(4x+5)^3$ where the tangent line is horizontal.

12. Find the slope of a line tangent to the circle $x^2+y^2=5x+4y$ at the point $P(5,4)$.

13. Use logarithmic differentiation to find $\frac{dy}{dx}$, where $y=\frac{e^{2x}}{(x^2-3)^2\ln\sqrt{x}}$.

14. Let $f(x) = 4x^3 + 15x^2 - 36x$. Find the intervals where f is concave up and concave down, respectively.

15. Find the limit: $\lim_{x\to\infty} \frac{3\sin x}{e^x}$.

16. Find the limit: $\lim_{x\to 5^-} \frac{x+6}{x-5}$.

17. Evaluate $\lim_{x\to -1} \left(\frac{x^{10}-1}{x+1}\right)$.

18. Evaluate $\lim_{x\to\infty} (x)^x$.

19. Evaluate $\int_1^4 \frac{x^3+\sqrt{x}}{x^2}\,dx$.

20. Evaluate $\int_1^{\sqrt{3}} \frac{6}{1+x^2}\,dx$.

21. Find the area of the region under the curve given in the following problem.
$y = x(x-1)^{\frac{1}{2}}$ over [1,3].

22. Find the average value of the following function:
$f(x) = \frac{2x}{2x+3}$ on [0,1].