Final Exam Math 1451 Version A Fall 2015

Please turn off **and** put away your cell phones. Calculators are **not** allowed. To receive any credit show all your work as described in class.

(1) Evaluate the following limits:

(i) $\lim_{x\to 1} \frac{x}{x^2+1}$ (ii) $\lim_{x\to\infty} \frac{x}{\sqrt{x^2-x}+x}$

(iii) $\lim_{x\to\infty} x^{-5}\ln(x)$ (iv) $\lim_{x\to\infty} x^{1/x}$

(2) For each function below find $\frac{dy}{dx}$:

(i) $y=\left(\frac{x^2+5}{x^2-5}\right)^3$ (ii) $y=\tan^{-1}(3x)$

(iii) $y=\ln\left(\sin^2(x)\right)$ (iv) $y=8x\sec^4\left(x^3\right)$

(3) Using implicit differentiation, find $\frac{dy}{dx}$ when $e^{xy}=3y^2-2\ln(x)$.

(4) Find the equation of the tangent line to the graph of the function $f(x)=\tan\left(\frac{x}{4}\right)$ at $x=\pi$.

(5) A 13 ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 6 ft/sec. How fast is the top of the ladder sliding down the wall at this point? *Include units.*

(6) Given the function $f(x)=\frac{2x^2-2x-12}{x^2-9}$, find the horizontal and vertical asymptotes.

(7) Given the function $f(x)=2x^4+16x^3-7$,
(i) determine the critical numbers of f.
(ii) determine whether each critical number is a relative maximum, relative minimum, or neither.

(8) Use the function $f(x)=3x^5-10x^4+x-1$ to
(i) find the inflections points of its graph.
(ii) determine where its graph is concave up.
(iii) determine where its graph is concave down.

(9) Find the following indefinite integrals:

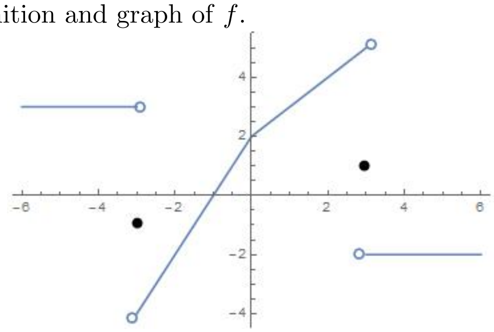
(i) $\int \frac{x^3+x^2\sin(x)-2}{x^2}\,dx$ (ii) $\int \frac{e^x}{e^x+1}\,dx$

(10) Evaluate the following definite integrals:

(i) $\int_{\frac{\pi}{4}}^{\frac{3\pi}{8}} \frac{3}{\cos^2\left(2x-\frac{\pi}{2}\right)}\,dx$ (ii) $\int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}}\,dx$

(11) For the subsequent questions, use the following definition and graph of f.

$$f(x)=\begin{cases} 3 & \text{if } x<-3; \\ -1 & \text{if } x=-3; \\ 2x+2 & \text{if } -3<x\le 0; \\ x+2 & \text{if } 0<x<3; \\ 1 & \text{if } x=3; \\ -2 & \text{if } 3<x. \end{cases}$$



(i) What is $\lim_{x\to -3} f(x)$? (ii) What is $\int_{-2}^{2} f(x)dx$?

(iii) What is $\lim_{h\to 0}\left(\frac{\int_0^{2+h} f(x)dx-\int_0^2 f(x)dx}{h}\right)$?