

Final 2015

Please turn off and put away your cell phones. Use of calculators is **not** allowed. To receive full credit show all your work and circle your final answer.

(1) Evaluate the following limits:

(i)  $\lim_{x \rightarrow \infty} \frac{5x^4 - 4x^2 + 1}{-x^4 + 10}$

(ii)  $\lim_{x \rightarrow 0^+} x^x$

(iii)  $\lim_{x \rightarrow \pi} \left( \frac{\cos(x/2)}{3x - 3\pi} \right)$

(2) For each function below find  $\frac{dy}{dx}$ :

(i)  $y = 2x^4 - e \ln(x) + e^x$

(ii)  $y = \sin^{-1}(3x)$

(iii)  $y = \cos(x) \sin(x)$

(iv)  $y = \frac{x}{\cos(x)}$

(3) Using implicit differentiation find  $y'$  when  $2x^2 + y^3 = 2$ .

(4) Find the equation for the line through the point  $(1, 2)$  tangent to curve  $y = x^2 + 1$

(5) A ladder 10 ft long rests against a vertical wall and is sliding down the wall at the rate of 2 ft/s at the instant the foot of the ladder is 6 ft from the base of the wall. At this instant, how fast is the foot of the ladder moving away from the wall?

(6) Given the function  $f(x) = \frac{3x^2 - 3x - 18}{x^2 + x - 2}$  find the horizontal and vertical asymptotes.

(7) Use the function  $f(x) = 3x^5 - 20x^3 + 4$  to answer the questions below:

(i) Where does  $f$  have relative maximums?      (ii) Where does  $f$  have relative minimums?

(8) Use the function  $g(x) = 2x^6 - 5x^4 + 2$  to answer the questions below:

(i) Where is  $g$  concave down?      (ii) Where is  $g$  concave up?

(iii) Where does  $g$  have inflection points?

(9) Find the indefinite integrals below:

(i)  $\int \left( \frac{x^2 + x^{-2}}{2} \right) dx$

(ii)  $\int (\cos^2(x) \sin(x)) dx$

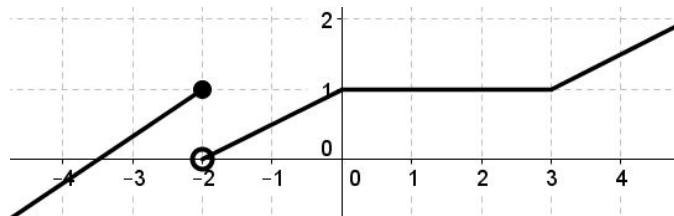
(10) Evaluate the following definite integrals:

(i)  $\int_0^{\frac{\pi}{3}} \tan(\theta) d\theta$

(ii)  $\int_1^e \left( \frac{\ln(x^4)}{x} \right) dx$

(11) For the subsequent questions use the following definition of  $f$ . The graph of the function is on the right.

$$f(x) = \begin{cases} \frac{2}{3}x + \frac{7}{3} & \text{if } x \leq -2; \\ \frac{1}{2}x + 1 & \text{if } -2 < x \leq 0; \\ 1 & \text{if } 0 < x \leq 3; \\ \frac{1}{2}x - \frac{1}{2} & \text{if } 3 < x. \end{cases}$$



(i) What is  $\lim_{x \rightarrow -2^-} f(x)$ ?

(ii) What is  $\int_{-2}^2 f(x) dx$ ?

(iii) What is  $\lim_{h \rightarrow 0} \left( \frac{\int_0^{4+h} f(x) dx - \int_0^4 f(x) dx}{h} \right)$ ?

(12) Using the Trapezoid rule approximate  $\int_0^4 x^2 dx$  with  $n = 4$ .

(13) Find the average value of  $f(x) = x^3$  on  $[0, 2]$ .