Final Exam Math 1451 Fall 2016

You may not use any printed/written material or electronic devices (including calculators and cell phones). For the Multiple Choice Problems, please choose only one answer. For the show-work answers, please use the space provided in the exam.

Note that the exam is double-sided. All your work must be included and submitted on this printout; no additional paper is collected. Please abide by the academic integrity rules: cheating, copying from another student, receiving or giving help on the exam will result in a score of 0 on the final - and will be reported to administrative offices, which will take appropriate action in the matter.

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Multiple Choice Part

Part D: Differentiation.
Evaluate the derivative of each of the following functions at the value $x = 0$, and choose the correct answer from the a)-d) group provided:

D1) $f(x) = x^3 + 2x^2 + 5x - 9$
Answer: a). 0; b). 5; c). -9; d). none of the above

D2) $f(x) = \arctan(7x) + x + 7$
Answer: a). 0; b). 7; c). 8; d). none of the above

D3) $f(x) = \cos x + \sin x$
Answer: a). 0; b). -1; c). 1; d). none of the above

D4) $f(x) = xe^{2x^2}$
Answer: a). 0; b). -1; c). 1; d). none of the above

D5) $f(x) = \frac{x}{2x^2 + 1} + x^{3/2} + x + 1$
Answer: a). 0; b). 1; c). 2; d). none of the above

D6) $f(x) = \arcsin(3x) + \pi/4$
Answer: a). 0; b). 1; c). 3; d). none of the above

D7) $f(x) = e \cdot \ln(\ln(x + e))$
Answer: a). 0; b). 1; c). e; d). none of the above

D8) $f(x) = \ln(e^{x^2+1}) + \pi/4$
Answer: a). 0; b). 1; c). 2; d). none of the above

D9) $f(x) = \tan x + x + 1$
Answer: a). 0; b). 1; c). 2; d). none of the above

D10) $f(x) = 2 \cos x \sin x$
Answer: a). 0; b). 1; c). 2; d). none of the above
Part I: Integration.
Evaluate the following definite integrals, and choose the correct answer from the a)-d) group provided:

I1) \[ \int_1^4 \frac{1}{2\sqrt{x}} \, dx \]
Answer: a). 0; b). 1; c). 2; d). none of the above

I2) \[ \int_0^1 3x^2 \, dx \]
Answer: a). 0; b). 1; c). 2; d). none of the above

I3) \[ \int_1^e \frac{x^2 + 1}{x} \, dx \]
Answer: a). 0; b). 1; c). 2; d). none of the above

I4) \[ \int_0^\pi \cos x + \sin x \, dx \]
Answer: a). 0; b). 1; c). 2; d). none of the above

I5) \[ \int_0^1 \frac{1}{1 + x^2} \, dx \]
Answer: a). 0; b). \( \pi \); c). \( \pi/4 \); d). none of the above

I6) \[ \int_0^1 \frac{4x}{1 + x^2} \, dx \]
Answer: a). 0; b). 2 \ln 2; c). 1; d). none of the above

I7) \[ \int_{\pi/6}^{\pi/2} \cot x \, dx \]
Answer: a). 0; b). \ln 2; c). \(-\ln 2\); d). none of the above

I8) \[ \int_0^1 2x \cdot e^{x^2} \, dx \]
Answer: a). 0; b). \( e \); c). \( e - 1 \); d). none of the above

I9) \[ \int_e^{e^2} \frac{1}{x \ln(x)} \, dx \]
Answer: a). 1; b). 2; c). \ln 2; d). none of the above

I10) \[ \int_0^\pi \sin x \cos x \, dx \]
Answer: a). 0; b). 1; c). 2; d). none of the above

Show Work Problems

I). A farmer has 400 feet of fencing to build a rectangular pen out on a field. Find the dimensions of the fenced plot that maximize the area. Justify your work using differential calculus, in order to receive proper credit, in the space provided:

II). Let \( f(x) = xe^x \) where \( x \) is a real number. Its 1st and 2nd derivatives are: \( f'(x) = \ldots \); \( f''(x) = \ldots \).
   (i) Draw a chart for the function \( f \), indicating the values \( x \) corresponding to critical points and inflection points, respectively.
   (ii) Using the chart, indicate the intervals where the function \( f \) is increasing, decreasing, concave down and concave up, respectively.
   (iii) Sketch the graph of the function \( f \).
III). Evaluate the following limits, if they exist. Box your final answers.

(i) \( \lim_{x \to 0} \frac{\sqrt{x + 4} - 2}{x} = \)

(ii) \( \lim_{x \to 0} (1 - x)^{\frac{3}{2}} = \)

Use this space and the back to work the Multiple Choice Problems, if and where needed