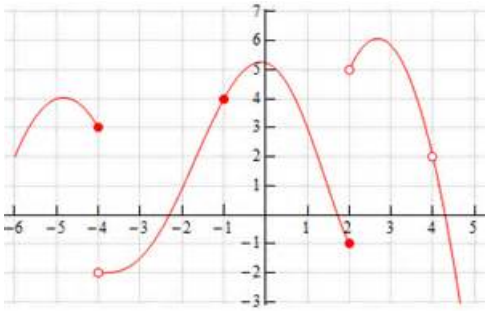
Final Exam Math 1451 Spring 2016

Please turn off **and** put away your cell phones. Calculators are **not** allowed. To receive any credit show your work as described in class. Present problems in your blue book in the order that they occur on the exam. Allow at least on full page for each problem. **This document is double-sided; there are problems on the back.**

(1) Evaluate the following limits, if they exist.

(i) The graph of the $f(x)$ is shown below. Find $\lim_{x\to 2^+} f(x)$, $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2} f(x)$.



(ii) $\lim_{x\to -3} \dfrac{x^3+2x^2-3x}{x^2+9}$

(iii) $\lim_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$

(iv) $\lim_{x\to 0} \left(\dfrac{1}{x^2} - \dfrac{\cos(x)}{x^2}\right)$

(v) $\lim_{x\to 0} (1+x)^{\frac{4}{x}}$

(2) Find the derivative of each of the following functions.

(i) $f(x) = 4x^7 - x^{-4} + x - 9$

(ii) $g(x) = \sin x - \cos x + \sec x$

(iii) $y = \dfrac{7}{(5x^3+7x-9)^5}$

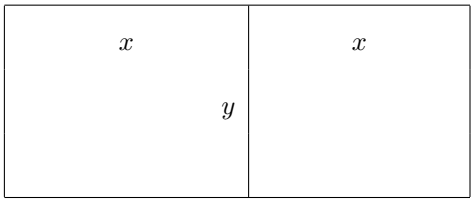
(iv) $s(t) = \left(\tan^{-1}(3t)\right)\sqrt{4t+3}$

(3) Using implicit differentiation, find $\dfrac{dy}{dx}$ when $e^{xy} + \ln y^2 = x$. *Your result should be in terms of both x and y.*

(4) Find an equation for the tangent line to the graph of $f(x) = \sin(2x)$ at the point $\left(\frac{\pi}{4}, 1\right)$.

(5) You and a friend are out skipping rocks at a local lake. However, on your next attempt, your stone does not skip, and instead sinks. As you stare at the place where the rock sunk, you notice that it caused a circular ripple in the water, and that the radius of the ripple is expanding at a rate of 5 in/sec. How fast is the area changing when the radius of the ripple is 20 inches? *Include units.*

(6) A landscape architect has 200 feet of fencing to enclose two adjacent rectangular fields of equal size (see figure). Find the dimensions of the fencing that maximize the area.



(7) Given $f(x) = \dfrac{3x^2-4}{x^2-9}$, find the horizontal and vertical asymptotes.

(8) Suppose the domain of f is all real numbers and the **derivative** is given by $f'(x) = 12x^5 - 12x^2$.

(i) Determine the critical numbers of f.

(ii) Determine the intervals where f is increasing.

(iii) Determine the intervals where f is decreasing.

(iv) Determine whether a relative minimum occurs at a critical number. If none exist, state none.

(v) Determine whether a relative maximum occurs at a critical number. If none exist, state none.

(9) Use the function $f(x) = e^{-2x^2}$
  (i) to find the inflection points of its graph;
  (ii) to determine where its graph is concave up;
  (iii) and to determine where its graph is concave down.

(10) Evaluate the following indefinite integrals:

(i) $\displaystyle\int \frac{x^4 - 4x^2 + 1}{x^2}\, dx$

(ii) $\displaystyle\int \left(5\cos(x) - 2\sec^2(x)\right)\, dx$

(iii) $\displaystyle\int \frac{x}{\sqrt{x+4}}\, dx$

(iv) $\displaystyle\int \frac{6x - 9}{(x^2 - 3x + 5)^3}\, dx$

(11) Evaluate the following definite integrals:

(i) $\displaystyle\int_0^{\frac{1}{2}} \frac{2}{1 + 4x^2}\, dx$

(ii) $\displaystyle\int_e^{e^8} \frac{1}{x(\ln(x))^{2/3}}\, dx$