

Please print your name clearly:.....

Final Exam Math 1451 Spring 2017

You may **not** use any printed/written material or electronic devices (including calculators and cell phones). For the Multiple Choice Problems, please choose only one answer. For the show-work answers, please use the space provided in the exam. **Note that the exam is double-sided. All your work must be included and submitted on this printout; no additional paper is collected.** Please abide by the academic integrity rules: cheating, copying from another student, receiving or giving help on the exam will result in a score of 0 on the final - and will be reported to administrative offices, which will take appropriate action in the matter.

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Multiple Choice Part

Part D: Differentiation.

Evaluate the derivative of each of the following functions **at the value** $x = 0$, and choose the correct answer from the a)-d) group provided:

D1) $f(x) = x^7 + 2x^5 + 5x^3 - 9x + 5$

Answer: a). 0; b). 5; c). -9 ; d). none of the above

D2) $f(x) = \cot(x + \frac{\pi}{2}) + x^3 + x^2 + 8$

Answer: a). -1; b). 1; c). 2 ; d). none of the above

D3) $f(x) = \arctan(2x) + 2$

Answer: a). 0; b). 2 ; c). 4 ; d). none of the above

D4) $f(x) = xe^x$

Answer: a). 0; b). -1 ; c). 1 ; d). none of the above

D5) $f(x) = \frac{x}{x+1}$

Answer: a). 0; b). 1 ; c). 2 ; d). none of the above

D6) $f(x) = \tan x + \pi/2$

Answer: a). 0; b). 1; c). 3 ; d). none of the above

D7) $f(x) = e \cdot \ln(x^2 + 1)$

Answer: a). 0; b). 1; c). e ; d). none of the above

D8) $f(x) = \ln(e^{(x^2+1)}) + \pi/4$

Answer: a). 0; b). 1; c). 2 ; d). none of the above

D9) $f(x) = \cos x + \sin x$

Answer: a). 0; b). 1; c). 2 ; d). none of the above

D10) $f(x) = 2 \cos x \sin x$

Answer: a). 0; b). 1; c). 2 ; d). none of the above

Part I: Integration.

Evaluate the following definite integrals, and choose the correct answer from the a)-d) group provided:

$$I1) \int_1^e \frac{1}{2x} dx$$

Answer: a). 1; b). 2; c). 1/2 ; d). none of the above

$$I2) \int_0^1 3x^2 + 2x + 1 dx$$

Answer: a). 1; b). 2; c). 3 ; d). none of the above

$$I3) \int_1^4 \frac{1}{2\sqrt{x}} dx$$

Answer: a). 0; b). 1; c). 2 ; d). none of the above

$$I4) \int_0^1 \frac{1}{1+x^2} dx$$

Answer: a). 0; b). $\pi/4$; c). π ; d). none of the above

$$I5) \int_{-1}^1 |x| dx$$

Answer: a). 0; b). 1/2; c). 1; d). none of the above

$$I6) \int_0^1 \frac{x}{1+x^2} dx$$

Answer: a). 1; b). $\ln 2$; c). $(\ln 2)/2$; d). none of the above

$$I7) \int_0^{\pi/2} \cos x dx$$

Answer: a). 0; b). 1 ; c). 2 ; d). none of the above

$$I8) \int_0^1 3x^2 \cdot e^{x^3} dx$$

Answer: a). 0; b). e ; c). $e - 1$; d). none of the above

$$I9) \int_1^e \frac{2 \ln x}{x} dx$$

Answer: a). 1; b). 2; c). $\ln 2$; d). none of the above

$$I10) \int_0^{2\pi} 2 \cos x \sin x dx$$

Answer: a). 0; b). 1; c). 2 ; d). none of the above

Show Work Problems

I). A rectangle $ABCD$ of sides $AB = x$ and $BC = y$, and diagonal $AC = 10$, is inscribed in a circle (i.e., vertices A, B, C, D belong to the circle). Find the values of x and y which maximize the area of this rectangle. What is the maximal area? Show and justify your work using differential calculus, in order to receive proper credit, in the space provided:

II). Let $f(x) = x^3 - x^2$ where x is a real number in the interval $[0,1]$.

Compute its 1st and 2nd derivatives: $f'(x) = \dots\dots\dots$; $f''(x) = \dots\dots\dots$

(i) Draw a chart below, for the function f , indicating the values x corresponding to critical points and inflection points, respectively.

(ii) Using the chart, indicate the intervals where the function f is increasing, decreasing, concave down and concave up, respectively.

(iii) Sketch the graph of the function f .

III). Evaluate the following limits, if they exist. Box your final answers.

(i) $\lim_{x \rightarrow 0} \frac{\sin(2x)}{3x} =$

(ii) $\lim_{x \rightarrow \infty} \frac{3x^3 + 2x + 1}{2x^3 + x^2 + x + 1} =$

Use this space and the back to work the Multiple Choice Problems, if and where needed