Part D: Differentiation.

Evaluate the derivative of each of the following functions at the value $x = 0$, and choose the correct answer from the a)-d) group provided:

D1) $f(x) = x^7 + 2x^5 + 5x^3 - 9x + 5$
Answer: a). 0; b). 5; c). -9; d). none of the above

D2) $f(x) = \cot(x + \frac{\pi}{2}) + x^3 + x^2 + 8$
Answer: a). -1; b). 1; c). 2; d). none of the above

D3) $f(x) = \arctan(2x) + 2$
Answer: a). 0; b). 2; c). 4; d). none of the above

D4) $f(x) = xe^x$
Answer: a). 0; b). -1; c). 1; d). none of the above

D5) $f(x) = \frac{x}{x + 1}$
Answer: a). 0; b). 1; c). 2; d). none of the above

D6) $f(x) = \tan x + \pi/2$
Answer: a). 0; b). 1; c). 3; d). none of the above

D7) $f(x) = e \cdot \ln(x^2 + 1)$
Answer: a). 0; b). 1; c). e; d). none of the above

D8) $f(x) = \ln(e^{(x^2+1)}) + \pi/4$
Answer: a). 0; b). 1; c). 2; d). none of the above

D9) $f(x) = \cos x + \sin x$
Answer: a). 0; b). 1; c). 2; d). none of the above

D10) $f(x) = 2 \cos x \sin x$
Answer: a). 0; b). 1; c). 2; d). none of the above
Part I: Integration.
Evaluate the following definite integrals, and choose the correct answer from the a)-d) group provided:

I1) \( \int_0^1 \frac{1}{2x} \, dx \)
Answer: a). 1; b). 2; c). 1/2; d). none of the above

I2) \( \int_0^1 3x^2 + 2x + 1 \, dx \)
Answer: a). 1; b). 2; c). 3; d). none of the above

I3) \( \int_1^4 \frac{1}{2\sqrt{x}} \, dx \)
Answer: a). 0; b). 1; c). 2; d). none of the above

I4) \( \int_0^1 \frac{1}{1 + x^2} \, dx \)
Answer: a). 0; b). \( \pi/4 \); c). \( \pi \); d). none of the above

I5) \( \int_{-1}^1 |x| \, dx \)
Answer: a). 0; b). 1/2; c). 1; d). none of the above

I6) \( \int_0^1 \frac{x}{1 + x^2} \, dx \)
Answer: a). 1; b). ln 2; c). (ln 2)/2; d). none of the above

I7) \( \int_0^{\pi/2} \cos x \, dx \)
Answer: a). 0; b). 1; c). 2; d). none of the above

I8) \( \int_0^1 3x^2 \cdot e^x \, dx \)
Answer: a). 0; b). e; c). \( e - 1 \); d). none of the above

I9) \( \int_1^e \frac{2\ln x}{x} \, dx \)
Answer: a). 1; b). 2; c). \( \ln 2 \); d). none of the above

I10) \( \int_0^{2\pi} 2 \cos x \sin x \, dx \)
Answer: a). 0; b). 1; c). 2; d). none of the above
Show Work Problems

I). A rectangle $ABCD$ of sides $AB = x$ and $BC = y$, and diagonal $AC = 10$, is inscribed in a circle (i.e., vertices $A, B, C, D$ belong to the circle). Find the values of $x$ and $y$ which maximize the area of this rectangle. What is the maximal area? Show and justify your work using differential calculus, in order to receive proper credit, in the space provided:

II). Let $f(x) = x^3 - x^2$ where $x$ is a real number in the interval $[0,1]$.
Compute its 1st and 2nd derivatives: $f'(x) = \ldots$; $f''(x) = \ldots$.

(i) Draw a chart below, for the function $f$, indicating the values $x$ corresponding to critical points and inflection points, respectively.

(ii) Using the chart, indicate the intervals where the function $f$ is increasing, decreasing, concave down and concave up, respectively.

(iii) Sketch the graph of the function $f$.

III). Evaluate the following limits, if they exist. Box your final answers.

(i) $\lim_{x \to 0} \frac{\sin(2x)}{3x} =$

(ii) $\lim_{x \to \infty} \frac{3x^3 + 2x + 1}{2x^3 + x^2 + x + 1} =$

Use this space and the back to work the Multiple Choice Problems, if and where needed.