# Calculus II Final Exam Spring 2012

*Calculators are not allowed on this exam. Work all questions completely. Show all work as described in class.*


1. Find the area of the region bounded by $y = 3x, \quad y = 0.1x, \quad x = 0.5, \quad$ and $\quad x = 2$.
2. Let the region $R$ of the plane be bounded by the curves $y = 3x^{2/3}, \quad y = 0, \quad$ and $\quad x = 1$.
   (a) Sketch the region $R$;
   (b) Find the volume of the solid formed when the region $R$ is revolved about the $y-$axis.
3. Find the arclength of the curve $y = 3(-1+6x)$ on the interval $[-12, 0]$.
4. Evaluate the following integrals:
   (a) $\int \dfrac{dx}{x(x-3)(x+1)}$; (b) $\int x^{-1}\ln(2x)\,dx$;
   (c) $\int e^{x^2} 3x\,dx$; (d) $\int \cos(3x)\sin(3x)\,dx$.

5. Evaluate the integral $\int_0^{\pi} \exp(2\sin x)\cos x\,dx$.

6. Evaluate $\int_{-\pi}^{\pi} x\cos(2x)dx$ by using integration by parts.

7. Find the $\lim_{n\to\infty} \dfrac{n^{2/3} - 3n + n^{1/2}}{n^{1/4} + 2n^{0.8} - 4n}$.

8. Does the series $\sum_{k=2}^{\infty} \left(\dfrac{2}{3}\right)^{k-2}$ converge? If so, find the sum. If not, explain why not.

9. (a) Find the limit $\lim_{n\to\infty} e^{\frac{1}{n}}$. (b) Does the series $\sum_{k=1}^{\infty} e^{\frac{1}{k}}$ converge? If no, explain why.

10. Do the following series converge or diverge? Specify the test used and show all the work needed to apply it.

$(a) \sum_{k=1}^{\infty} \dfrac{1}{k^{4/3}};$

$(b) \sum_{k=1}^{\infty} \dfrac{1}{100k}.$

11. Does the series $\sum_{k=1}^{\infty} \dfrac{(-1)^k}{2+k}$ converge absolutely, converge conditionally, or diverge?

12. Does the $\lim_{N\to\infty}\sum_{k=1}^{N}\frac{2^k}{k!}$ exist? Explain completely.

13. Find the radius and the interval of the convergence of the power series $\sum_{k=1}^{\infty}(x+1)^k 5^k$ .

14. Let $f(x)=\ln(3x)$ .

(a) Find the first two terms of the Taylor series of the function $f(x)$ about $x=1$.

(b) Let $T_2(f,x)$ be the sum of first two terms in the Taylor series of f about x=1 that you found in part

(c) Find the $\lim_{x\to\infty}\frac{f(x)}{T_2(f,x)}$.